Chapter 5: Functions and Graphs

5.3 Inverse Functions

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Learning Outcomes

(a) Show whether a function has an inverse and find the inverse of the function

* Use algebraic or graphical approach.

* Emphasize that the inverse exists only for one-to-one and onto functions.

(b) Compute the inverse of a function.

$$f \cdot g(x) = g \cdot f(x) = x \;.\; implies\ f\ inverse\ of\ g$$

(c) Identify the domain and range of an inverse function.

(d) Sketch the graph of the function f and its inverse f^{-1} on the same axes.

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Bloom: Understanding

Inverse functions

The inverse of a function f exists if and only if f is a one-to-one function.

**Refer SDL 5.1 to determine one-to-one function using algebraic method or horizontal line test.*

Domain and Range of Inverse Function

Domain of f^{-1} = Range of f

Range of f^{-1} = Domain of f

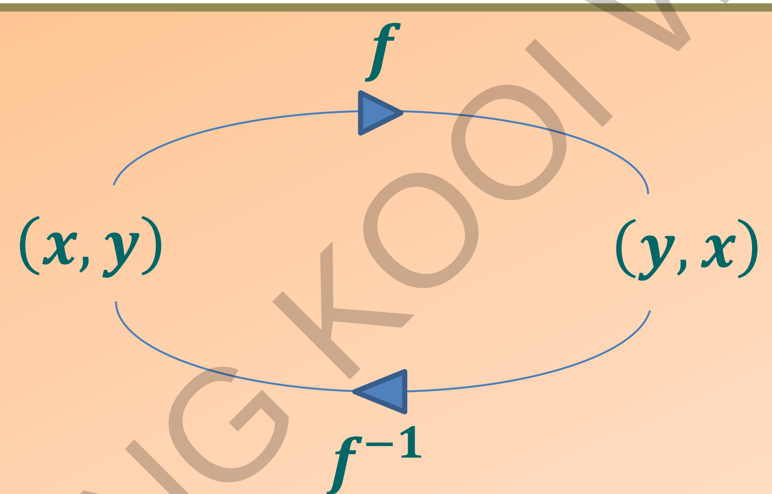
$$f^{-1}[f(x)] = f\left[f^{-1}(x)\right] = x$$

Bloom: Remembering

Bloom: Understanding

Inverse Functions

Relationship between graphs of a function and its inverse



The graph of f^{-1} is obtained by reflecting the graph of f in the line $y = x$. The points (x, y) and (y, x) are at the same distances from the line $y = x$.

Bloom: Remembering
Bloom: Understanding

Example

1. The function f is defined by $f: x \to 2x - 1, x \in R$.
   (a) Show that f is a one-to-one function.
   (b) Find f^{-1}.
   (c) Verify that $f\left(f^{-1}(x)\right) = x$.

2. Find the inverse for the function $f(x) = x^2 + x - 2, x \geq -\frac{1}{2}$ and state the domain and the range for the inverse function. Sketch the graph of $y = f(x)$ and $y = f^{-1}(x)$ in the same diagram.

Bloom: Understanding

Solution

1. (a) $f(x_1) = f(x_2)$

Let $f(x_1) = f(x_2)$.

$2x_1 - 1 = 2x_2 - 1$

$x_1 = x_2$

Hence f is one-to-one function.

(b) $f(x) = 2x - 1 = y$

Let $f(x) = y$.

$\therefore \quad x = \dfrac{y+1}{2}$

Rearranging to let x in term of y .

$\therefore \quad f^{-1}(y) = x = \dfrac{y+1}{2}$

Hence, $f^{-1}(x) = \dfrac{x+1}{2}$

Bloom: Understanding

Solution

1. (c)

$$f\left(f^{-1}(x)\right) = f\left(\frac{x+1}{2}\right)$$

$$= 2\left(\frac{x+1}{2}\right) - 1$$

$$= x$$

Composite function definition.

2.

$$f(x) = x^2 + x - 2, x \geq -\frac{1}{2}$$

$$f(x) = x^2 + x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 2$$

$$= \left(x + \frac{1}{2}\right)^2 - \frac{9}{4}$$

Completing the square.

Minimum point $= \left(-\frac{1}{2}, -\frac{9}{4}\right)$.

Bloom: Understanding

Solution

2. ***Continue…***

$$f\left(f^{-1}(x)\right) = x$$

$$\left(f^{-1}(x) + \frac{1}{2}\right)^2 - \frac{9}{4} = x$$

$$f^{-1}(x) = -\frac{1}{2} \pm \sqrt{x + \frac{9}{4}}$$

Since, $R_{f^{-1}} = D_f = \left[-\frac{1}{2}, \infty\right)$

$$\therefore f^{-1}(x) = -\frac{1}{2} + \sqrt{x + \frac{9}{4}}$$

From minimum point,

Domain f $= \left[-\frac{1}{2}, \infty\right)$

Range f $= \left[-\frac{9}{4}, \infty\right)$

$$D_{f^{-1}} = R_f$$
$$R_{f^{-1}} = D_f$$

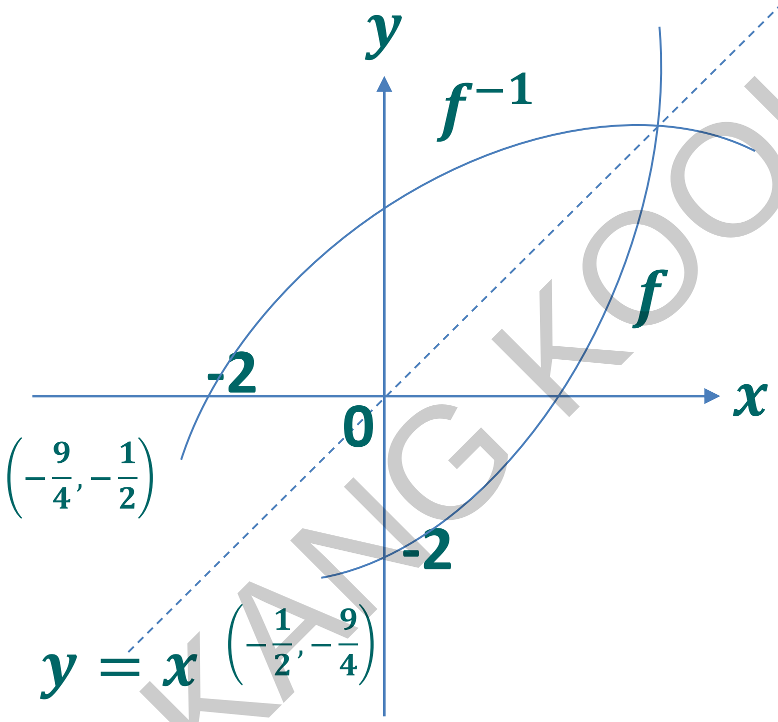
$$\therefore D_{f^{-1}} = \left[-\frac{9}{4}, \infty\right)$$

$$R_{f^{-1}} = \left[-\frac{1}{2}, \infty\right)$$

Bloom: Understanding

Solution

2. ***Continue…***



Self-check

1. The function f is defined by $f: x \to x^3 - 1, x \in R$.

   (a) Show that f is a one-to-one function.

   (b) Find f^{-1} .

   (c) Verify that $f\left(f^{-1}(x)\right) = x$.

2. Find the inverse for the function $f(x) = x^2 + 2x + 3, x \geq -1$ and state the domain and the range for the inverse function. Sketch the graph of $y = f(x)$ and $y = f^{-1}(x)$ in the same diagram.

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Bloom: Applying

Answer Self-check

1. (b) $f^{-1}(x) = (x+1)^{\frac{1}{3}}$

2. $f^{-1}(x) = -1 + \sqrt{x-2}$

$D_{f^{-1}} = [2, \infty)$

$R_{f^{-1}} = [-1, \infty)$

Bloom: Applying

Summary

Inverse Function f^{-1}

- The inverse of a function f exists if and only if f is a one-to-one function.
- $D_{f^{-1}} = R_f$
- $R_{f^{-1}} = D_f$
- The graphs of f and f^{-1} are reflecting on the line $y = x$.
- $f^{-1}[f(x)] = f\left[f^{-1}(x)\right] = x$

Key Terms

- Inverse Functions
- Domain
- Range
- Composite Functions
- Function