Chapter 5: Functions and Graphs

5.2 Composite Functions

Prepared by: Kang Kooi Wei

Learning Outcomes

(a) Represent a composite function using an arrow diagram

(b) Find composite function.

** Include repeated composite functions such as $f \cdot g$ $f \cdot g \cdot h$.*

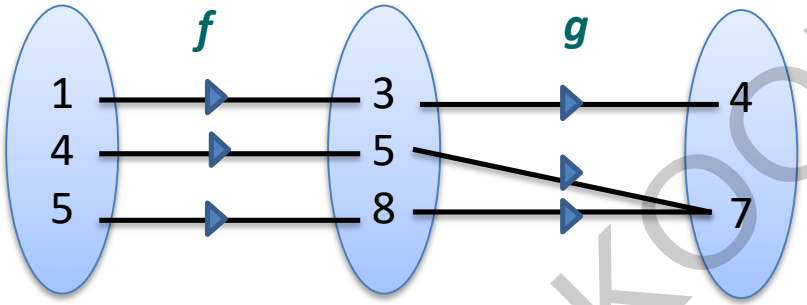
(c) Find one of the functions when the composite and the other functions are given.

** Exclude domain, range and existence of composite functions.*

Bloom: Understanding

Composite functions

Arrow diagram represent composite functions



The arrow diagram shows two functions f and g. Find the value of

(a) $(g \cdot f)(1)$

(b) $(g \cdot f)(4)$

(c) $(g \cdot f)(5)$

Bloom: Remembering

Bloom: Understanding

KangKooiWei@KMK

Composite functions

Solution:

(a)

$$f(1) = 3$$
$$g(3) = 4$$
$$(g \cdot f)(1) = 4$$

(b)

$$f(4) = 5$$
$$g(5) = 7$$
$$(g \cdot f)(4) = 7$$

(c)

$$f(5) = 8$$
$$g(8) = 7$$
$$(g \cdot f)(5) = 7$$

Bloom: Remembering
Bloom: Understanding

Example

Given $f(x) = 2x$ and $g(x) = x + 1$. Find the composite functions of $g \cdot f$ and $f \cdot g$.

Solution:

$$(g \cdot f)(x) = g(2x)$$

Substituting $f(x) = 2x$.

$$= (2x) + 1$$

Replacing x with $2x$ in the function $f(x)$.

$$= 2x + 1$$

$$(f \cdot g)(x) = f(x + 1)$$

Substituting $g(x) = x + 1$.

$$= 2(x + 1)$$

Replacing x with $x + 1$ in the function $g(x)$.

Bloom: Understanding

Example

Given $f(x) = 2x$ and $g(x) = x + 1$. Find the composite functions of $g \cdot f$ and $f \cdot g$.

Solution:

$$(g \cdot f)(x) = g(2x)$$

Substituting $f(x) = 2x$.

$$= (2x) + 1$$

Replacing x with $2x$ in the function $f(x)$.

$$= 2x + 1$$

$$(f \cdot g)(x) = f(x + 1)$$

Substituting $g(x) = x + 1$.

$$= 2(x + 1)$$

Replacing x with $x + 1$ in the function $g(x)$.

Bloom: Understanding

Example

Three functions f , g and h are defined as follows.

$f(x) = x^3$, $g(x) = 8 - 3x$ and $h(x) = 2x$

Find

(a) $(f \cdot g^2)(x)$

(b) $(f \cdot g \cdot h)(x)$

Bloom: Understanding

Solution

(a) $(f \cdot g^2)(x) = f[g(g(x))]$ Using definition of $g^2(x) = g[g(x)]$.

$= f[g(8 - 3x)]$ Substituting $g(x) = 8 - 3x$.

$= f[8 - 3(8 - 3x)]$ Replacing x with 8-3x in the function $g(x)$.

$= f[9x - 16]$

$= (9x - 16)^3$ Replacing x with $9x - 16$ in the function $f(x)$.

Bloom: Understanding

Solution

(b) $(f \cdot g \cdot h)(x) = f[g(h(x))]$ — Using definition of $f \cdot g \cdot h$.

$= f[g(2x)]$ — Substituting $h(x) = 2x$.

$= f[8 - 3(2x)]$ — Replacing x with $2x$ in the function $g(x)$.

$= f[8 - 6x]$

$= (8 - 6x)^3$ — Replacing x with $8 - 6x$ in the function $f(x)$.

Bloom: Understanding

Example

Given $(f \cdot g)(x) = 3x + 5$ and $g(x) = 2x - 1$, find $f(x)$.

Solution:

$$(f \cdot g)(x) = 3x + 5$$

$$f[g(x)] = 3x + 5$$

$$f[2x - 1] = 3x + 5$$

Substituting $g(x) = 2x - 1$.

Let $y = 2x - 1$

Representing $2x - 1$ by y.

$$x = \frac{y + 1}{2}$$

Expressing x in terms of y.

Bloom: Understanding

Solution

$$f(y) = 3\left(\frac{y+1}{2}\right) + 5$$

$$= \frac{3y + 3 + 10}{2}$$

$$= \frac{3y + 13}{2}$$

$$f(x) = \frac{3x + 13}{2}$$

Substituting $x = \frac{y+1}{2}$ into the function $f[2x - 1] = 3x + 5$.

Replacing y with x to obtain $f(x)$.

Bloom: Understanding

Example

Find $f(x)$ such that $(g \cdot f)(x) = x + 2$ and $g(x) = \sqrt{x - 1}$.

Solution:

$$(g \cdot f)(x) = x + 2$$

$$g[f(x)] = x + 2$$

$$\sqrt{f(x) - 1} = x + 2$$

Replacing x with $f(x)$ in the function $g(x)$.

$$f(x) - 1 = (x + 2)^2$$

Squaring both sides.

$$f(x) = (x + 2)^2 + 1$$

Solving for $f(x)$.

Bloom: Understanding

Self-check

(1) The functions f and g are defined by

$$f: x \to 3x + 1, x \in R$$

$$g: x \to x - 4, x \in R$$

Find

(a) $(f \cdot g)(x)$

(b) $(g \cdot f)(x)$

Bloom: Applying

Self-check

(2) Find the function g such that $f \cdot g(x) = \sqrt{\frac{x+1}{x-1}}$ if

$f(x) = \sqrt{x+1}$.

(3) Find the function f such that $f \cdot g(x) = 32x^6 + 16x^3 + 5$ if $g(x) = 4x^3 + 1$.

Bloom: Applying

Answer Self-check

(1) (a) $3x - 11$ (b) $3(x - 1)$

(2) $g(x) = \dfrac{2}{x - 1}$

(3) $f(x) = 2x^2 + 3$

Bloom: Applying

Summary

Composite Functions

$$(f \cdot g)(x) = f[g(x)]$$

- Determine $f \cdot g$ or $g \cdot f$.
- Determine the function f or g from a composite function.

Key Terms

- Composite Functions
- Arrow diagram
- Function