

QS015/2
Mathematics
Paper 2
Semester I
Session 2014/2015
2 hours

QS015/2
Matematik
Kertas 2
Semester I
Sesi 2014/2015
2 jam



KEMENTERIAN
PENDIDIKAN
MALAYSIA

BAHAGIAN MATRIKULASI
MATRICULATION DIVISION

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **15** halaman bercetak.

This question paper consists of 15 printed pages.

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

LIST OF MATHEMATICAL FORMULAE

Limit

$$\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$$

$$\lim_{h \rightarrow 0} \frac{1 - \cos h}{h} = 0$$

Differentiation

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

If $y = g(t)$ and $x = f(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{d^2 y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}}$$

Sphere	$V = \frac{4}{3} \pi r^3$	$S = 4 \pi r^2$
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Right circular cone	$V = \frac{1}{3} \pi r^2 h$	$S = \pi r s$
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Right circular cylinder	$V = \pi r^2 h$	$S = 2 \pi r h$
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- 1 Given that $(x-2)$ is a factor of the polynomial $f(x) = ax^3 - 10x^2 + bx - 2$ where a and b are real numbers. If $f(x)$ is divided by $(x+1)$ the remainder is -24 , find the values of a and b . Hence, find the remainder when $f(x)$ is divided by $(2x+1)$.

[6 marks]

- 2 Solve the equation $2\cos^2 x - 1 = \sin x$ for $0 \leq x \leq 2\pi$. Give your answer in terms of π .

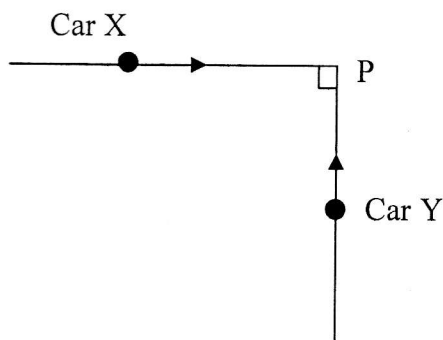
[6 marks]

- 3 Find the relative extremum of the curve $y = x^3 - 4x^2 + 4x$.

[6 marks]

- 4 Car X is travelling east at a speed of 80 km/h and car Y is travelling north at 100 km/h as shown in the diagram below. Obtain an equation that describes the rate of change of the distance between the two cars.

Hence, evaluate the rate of change of the distance between the two cars when car X is 0.15 km and car Y is 0.08 km from P.



[7 marks]

- 5 Expand $(x+a)(x+b)^2$, a and b are real numbers with $b > 0$. Hence, find the values of a and b if $(x+a)(x+b)^2 = x^3 - 3x - 2$.

Express $\frac{x^4 - 4x^2 + 5x - 1}{x^3 - 3x - 2}$ in the form of partial fractions.

[12 marks]

- 6 (a) Express $\sin 6x - \sin 2x$ in a product form. Hence, show that $\sin 6x - \sin 2x + \sin 4x = 4 \cos 3x \sin 2x \cos x$.

[6 marks]

- (b) Use the result in (a) to solve $\sin 6x - \sin 2x + \sin 4x = \sin 2x \cos x$ for $0 \leq x \leq 180^\circ$.

[7 marks]

- 7 Find the limit of the following, if it exists.

(a) $\lim_{x \rightarrow -3} \frac{x+3}{x^3+27}$.

[3 marks]

(b) $\lim_{x \rightarrow -\infty} \frac{2x-1}{\sqrt{x^2-9}}$.

[3 marks]

(c) $\lim_{x \rightarrow 4} \frac{x^2-3x-4}{\sqrt{x}-2}$.

[4 marks]

8 Given that $f(x) = \begin{cases} 1+e^x, & x \leq 0 \\ \frac{x+6}{3-x}, & 0 < x \leq 4 \\ C, & x > 4 \end{cases}$

where C is a constant.

- (a) Determine whether $f(x)$ is continuous at $x = 0$.

[5 marks]

- (b) Given that $f(x)$ is discontinuous at $x = 4$, determine the values of C .

[3 marks]

- (c) Find the vertical asymptote of $f(x)$.

[4 marks]

- 9 Consider the parametric equations of the curve

$$x = \cos^3 \theta \text{ and } y = \sin^3 \theta, \quad 0 < \theta \leq 2\pi.$$

- (a) Find $\frac{dy}{dx}$ and express your answer in terms of θ .

[4 marks]

- (b) Find the value of $\frac{dy}{dx}$ if $x = \frac{\sqrt{2}}{4}$.

[4 marks]

- (c) Show that $\frac{d^2y}{dx^2} = \frac{1}{3 \cos^4 \theta \sin \theta}$.

Hence, calculate $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

[5 marks]

- 10 (a) Use the first principle to find the derivative of $g(x) = \sqrt{1-x}$.

[5 marks]

- (b) Given that $e^y + xy + \ln(1+2x) = 1$, $x \geq 0$.

Show that $(e^y + x) \frac{d^2y}{dx^2} + e^y \left(\frac{dy}{dx} \right)^2 + 2 \frac{dy}{dx} - \frac{4}{(1+2x)^2} = 0$.

Hence, find the value of $\frac{d^2y}{dx^2}$ at the point $(0,0)$.

[10 marks]

END OF QUESTION PAPER