KEMENTERIAN PENDIDIKAN MALAYSIA

QS 015/1

Matriculation Programme Examination

Semester 1

Session 2012/2013

1. Find the value of x which satisfies the equation

$$log_2(5-x) - log_2(x-2) = 3 - log_2(1+x)$$

2. Determine the solution set of the inequality

$$\frac{1}{2x-1} < \frac{1}{x+2}$$

3. Given $k+2, k-4, k-7$ are the first three terms of a geometric series. Determine the value of k. Hence, find the sum to infinity of the series.

4. Given a complex number $z = 1 - \sqrt{3}i$. Determine the value of k if $\overline{z^2} = k\frac{1}{z}$.

5. (a) Matrix M is given as $\begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix}$. Show that $M^2 = 7M - 8I$, where I is the 2 x 2 identity matrix. Deduce that $M^{-1} = \frac{7}{8}I - \frac{1}{8}M$.

   (b) Given matrix $A = \begin{bmatrix} p+1 & -1 & 1 \\ 3 & 2 & 4 \\ -1 & 0 & p+2 \end{bmatrix}$ and $|A| = 27$. Find the value of p, where p is an integer.

6. The functions f and g are defined as $f(x) = \frac{3x+4}{x-2}$, $x \neq 2$ and $g(x) = 3 - x$.

   (a) Find $f^{-1}(x)$ and $g^{-1}(x)$.

   (b) Evaluate $(f \circ g^{-1})(3)$.

   (c) If $(g \circ f^{-1})(k) = \frac{2}{3}$, find the value of k.

7. (a) Solve $|x^2 - x - 3| = 3$.

   (b) Find the solution set of the inequality $\frac{2x^2+9x-4}{x+2} < 4$.

8. The first four terms of a binomial expansion $(1+ax)^n$ is

$$1 + x - \frac{1}{2}x^2 + px^3 + \cdots$$

Find

(a) the values of a and n where $n \neq 0$.

(b) the value of p Hence, by substituting $x = \frac{1}{4}$, show that $\sqrt{\frac{3}{2}}$ is approximately equal to $\frac{157}{128}$.

9. Given $f(x) = \ln(2x+3)$ and $g(x) = \frac{e^x - 3}{2}$.

(a) Show that $f(x)$ is a one-to-one function algebraically.

(b) Find $(f \circ g)(x)$ and $(g \circ f)(x)$. Hence, state the conclusion about the results.

(c) Sketch the graphs of $f(x)$ and $g(x)$ on the same axes. Hence, state the domain and range of $f(x)$.

10. Given

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 5 & 4 \\ 3 & 1 & 4 \end{bmatrix}$$

(a) Find the determinant of matrix A.

(b) Find the minor, cofactor and adjoint of matrix A.

(c) Given $A(adjoint(A)) = |A|I$ where I is 3 x 3 identity matrix, show that $A^{-1} = \frac{1}{|A|} adjoint(A)$. Hence, find A^{-1}.

(d) By using A^{-1} in part (c), solve the following simultaneous equations.

$$2x + 2y + 3z = 49$$
$$x + 5y + 4z = 74$$
$$3x + y + 4z = 49$$

END OF QUESTION PAPER

1. Find the value of x which satisfies the equation

$$log_2(5-x) - log_2(x-2) = 3 - log_2(1+x)$$

SOLUTION

$$log_2(5-x) - log_2(x-2) = 3 - log_2(1+x)$$
$$log_2\frac{(5-x)}{(x-2)} = 3 - log_2(1+x)$$
$$log_2\frac{(5-x)}{(x-2)} + log_2(1+x) = 3$$
$$log_2\frac{(5-x)(1+x)}{(x-2)} = 3$$

$log_a b = c \rightarrow b = a^c$

$$\frac{(5-x)(1+x)}{(x-2)} = 2^3$$
$$\frac{(5-x)(1+x)}{(x-2)} = 8$$
$$(5-x)(1+x) = 8(x-2)$$
$$5 + 5x - x - x^2 = 8x - 16$$
$$5 + 4x - x^2 = 8x - 16$$
$$0 = x^2 + 8x - 4x - 16 - 5$$
$$x^2 + 8x - 4x - 16 - 5 = 0$$
$$x^2 + 4x - 21 = 0$$
$$(x+7)(x-3) = 0$$

$(x+7) = 0$ or $(x-3) = 0$

$x = -7$ or $x = 3$

<u>*Checking*</u>

$$log_2(5-x) - log_2(x-2) = 3 - log_2(1+x)$$

When $x = -7$

$$log_2[5-(-7)] - log_2(-7-2) = 3 - log_2(1-7)$$
$$x \neq -7$$

When $x = 3$

$$log_2(5-3) - log_2(3-2) = 3 - log_2(1+3)$$
$$log_2(2) - log_2(1) = 3 - log_2(4)$$
$$1 - 0 = 3 - 2$$
$$1 = 1 \surd$$

$\therefore x = 3$

2. Determine the solution set of the inequality

$$\frac{1}{2x-1} < \frac{1}{x+2}$$

SOLUTION

$$\frac{1}{2x-1} < \frac{1}{x+2}$$

$$\frac{1}{2x-1} - \frac{1}{x+2} < 0$$

$$\frac{1(x+2) - 1(2x-1)}{(2x-1)(x+2)} < 0$$

$$\frac{x+2-2x+1}{(2x-1)(x+2)} < 0$$

$$\frac{-x+3}{(2x-1)(x+2)} < 0$$

$-x+3=0$ $\qquad$ $2x-1=0$ $\qquad$ $x+2=0$

$x=3$ $\qquad$ $x=\frac{1}{2}$ $\qquad$ $x=-2$

	$(-\infty,-2)$	$\left(-2,\frac{1}{2}\right)$	$\left(\frac{1}{2},3\right)$	$(3,\infty)$
$-x+3$	+	+	+	-
$2x-1$	-	-	+	+
$x+2$	-	+	+	+
$\frac{-x+3}{(2x-1)(x+2)}$	+	-	+	-

$$\therefore \textit{The solution is} \left(-2,\frac{1}{2}\right) \cup (3,\infty)$$

3. Given $k+2, k-4, k-7$ are the first three terms of a geometric series. Determine the value of k. Hence, find the sum to infinity of the series.

SOLUTION

$k+2, k-4, k-7 \ldots$

$$\frac{k-4}{k+2} = \frac{k-7}{k-4}$$

$(k-4)(k-4) = (k-7)(k+2)$

$k^2 - 4k - 4k + 16 = k^2 + 2k - 7k - 14$

$k^2 - 8k + 16 = k^2 - 5k - 14$

$16 + 14 = -5k + 8k$

$3k = 30$

$k = 10$

When $k = 10$, the sequence is

$10+2, 10-4, 10-7 \ldots$

$12, 6, 3 \ldots$

$a = 12,\qquad r = \frac{6}{12} = \frac{1}{2}$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{12}{1-\left(\frac{1}{2}\right)}$$

$$S_\infty = \frac{12}{\left(\frac{1}{2}\right)}$$

$S_\infty = 24$

4. Given a complex number $z = 1 - \sqrt{3}i$. Determine the value of k if $\overline{z^2} = k\frac{1}{\bar{z}}$.

SOLUTION

$z = 1 - \sqrt{3}i \qquad\qquad \bar{z} = 1 + \sqrt{3}i$

$$\overline{z^2} = k\frac{1}{\bar{z}}$$

$$\overline{\left(1 - \sqrt{3}\imath\right)^2} = k\left(\frac{1}{1 + \sqrt{3}i}\right)$$

$$\overline{\left(1 - \sqrt{3}\imath\right)\left(1 - \sqrt{3}\imath\right)} = \frac{k}{1 + \sqrt{3}i}$$

$$\overline{1 - 2\sqrt{3}\imath + 3\imath^2} = \frac{k}{1 + \sqrt{3}i}$$

$$\overline{1 - 2\sqrt{3}\imath + 3(-1)} = \frac{k}{1 + \sqrt{3}i}$$

$$\overline{1 - 2\sqrt{3}\imath - 3} = \frac{k}{1 + \sqrt{3}i}$$

$$\overline{-2 - 2\sqrt{3}\imath} = \frac{k}{1 + \sqrt{3}i}$$

$$-2 + 2\sqrt{3}i = \frac{k}{1 + \sqrt{3}i}$$

$$k = \left(-2 + 2\sqrt{3}i\right)\left(1 + \sqrt{3}i\right)$$

$$k = -2 - 2\sqrt{3}i + 2\sqrt{3}i + 6i^2$$

$$k = -2 + 6(-1)$$

$$k = -8$$

5. (a) Matrix M is given as $\begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix}$. Show that $M^2 = 7M - 8I$, where I is the 2 x 2 identity matrix. Deduce that $M^{-1} = \frac{7}{8}I - \frac{1}{8}M$.

(b) Given matrix $A = \begin{bmatrix} p+1 & -1 & 1 \\ 3 & 2 & 4 \\ -1 & 0 & p+2 \end{bmatrix}$ and $|A| = 27$. Find the value of p, where p is an integer.

<u>SOLUTION</u>

<u>5(a)</u>

$$M = \begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 9+4 & -3-4 \\ -12-16 & 4+16 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 13 & -7 \\ -28 & 20 \end{bmatrix}$$

$$7M - 8I = 7\begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix} - 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$7M - 8I = \begin{bmatrix} 21 & -7 \\ -28 & 28 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

$$7M - 8I = \begin{bmatrix} 13 & -7 \\ -28 & 20 \end{bmatrix}$$

$$\therefore M^2 = 7M - 8I$$

$$M^2 = 7M - 8I$$

$$M^2 - 7M = -8I$$

$$M(M - 7I) = -8I$$

$$M^{-1} = -\frac{1}{8}(M - 7I)$$

$$M^{-1} = -\frac{1}{8}M + \frac{7}{8}I$$

$$M^{-1} = \frac{7}{8}I - \frac{1}{8}M$$

_5(b)_

$$A = \begin{bmatrix} p+1 & -1 & 1 \\ 3 & 2 & 4 \\ -1 & 0 & p+2 \end{bmatrix}$$

$$|A| = 27$$

$$|A| = +(p+1)\begin{vmatrix} 2 & 4 \\ 0 & p+2 \end{vmatrix} - (-1)\begin{vmatrix} 3 & 4 \\ -1 & p+2 \end{vmatrix} + (1)\begin{vmatrix} 3 & 2 \\ -1 & 0 \end{vmatrix}$$

$$|A| = +(p+1)[(2p+4)-(0)] + [(3p+6)-(-4)] + [(0)-(-2)]$$

$$|A| = (p+1)(2p+4) + (3p+6+4) + 2$$

$$|A| = 2p^2 + 4p + 2p + 4 + 3p + 6 + 4 + 2$$

$$|A| = 2p^2 + 9p + 16$$

$$2p^2 + 9p + 16 = 27$$

$$2p^2 + 9p - 11 = 0$$

$$(2p+11)(p-1) = 0$$

$$p = -\frac{11}{2} \qquad p = 1$$

Since p is an integer, $\therefore p = 1$

6. The functions f and g are defined as $f(x) = \frac{3x+4}{x-2}$, $x \neq 2$ and $g(x) = 3 - x$.

(a) Find $f^{-1}(x)$ and $g^{-1}(x)$.

(b) Evaluate $(f \circ g^{-1})(3)$.

(c) If $(g \circ f^{-1})(k) = \frac{2}{3}$, find the value of k.

<u>SOLUTION</u>

<u>6(a)</u>

$f(x) = \dfrac{3x+4}{x-2}$ $y = \dfrac{3x+4}{x-2}$ $y(x-2) = 3x+4$ $xy - 2y = 3x + 4$ $xy - 3x = 4 + 2y$ $x(y-3) = 4 + 2y$ $x = \dfrac{4+2y}{y-3}$ $f^{-1}(x) = \dfrac{4+2x}{x-3}$	$g(x) = 3 - x$ $y = 3 - x$ $x = 3 - y$ $g^{-1}(x) = 3 - x$

<u>6(b)</u>

$f(x) = \frac{3x+4}{x-2}$ $\qquad$ $f^{-1}(x) = \frac{4+2x}{x-3}$

$g(x) = 3 - x$ $\qquad$ $g^{-1}(x) = 3 - x$

$$(f \circ g^{-1})(x) = f[g^{-1}(x)]$$

$$= f[3 - x]$$

$$= \frac{3(3-x)+4}{(3-x)-2}$$

$$= \frac{9 - 3x + 4}{3 - x - 2}$$

$$= \frac{13 - 3x}{1 - x}$$

$$(f \circ g^{-1})(3) = \frac{13 - 3(3)}{1 - (3)}$$

$$(f \circ g^{-1})(3) = \frac{13 - 9}{-2}$$

$$(f \circ g^{-1})(3) = -2$$

6(c)

$f(x) = \frac{3x+4}{x-2}$ $\quad\quad$ $f^{-1}(x) = \frac{4+2x}{x-3}$

$g(x) = 3 - x$ $\quad\quad$ $g^{-1}(x) = 3 - x$

$$(g \circ f^{-1})(k) = \frac{2}{3}$$

$$(g \circ f^{-1})(k) = g[f^{-1}(k)]$$

$$= g\left[\frac{4 + 2k}{k - 3}\right]$$

$$= 3 - \left(\frac{4 + 2k}{k - 3}\right)$$

$$3 - \left(\frac{4 + 2k}{k - 3}\right) = \frac{2}{3}$$

$$\frac{4 + 2k}{k - 3} = 3 - \frac{2}{3}$$

$$\frac{4 + 2k}{k - 3} = \frac{7}{3}$$

$$3(4 + 2k) = 7(k - 3)$$

$$12 + 6k = 7k - 21$$

$$k = 33$$

7. (a) Solve $|x^2 - x - 3| = 3$.

(b) Find the solution set of the inequality $\frac{2x^2+9x-4}{x+2} < 4$.

SOLUTION

7(a)

$|x^2 - x - 3| = 3$

$x^2 - x - 3 = 3$ or $x^2 - x - 3 = -3$

$x^2 - x - 6 = 0$ or $x^2 - x = 0$

$(x-3)(x+2) = 0$ or $x(x-1) = 0$

$x = 3 \text{ or } x = -2$ or $x = 0 \text{ or } x = 1$

7(b)

$$\frac{2x^2+9x-4}{x+2} < 4$$

$$\frac{2x^2+9x-4}{x+2} - 4 < 0$$

$$\frac{(2x^2+9x-4) - 4(x+2)}{x+2} < 0$$

$$\frac{2x^2+9x-4-4x-8}{x+2} < 0$$

$$\frac{2x^2+5x-12}{x+2} < 0$$

$$\frac{(2x-3)(x+4)}{x+2} < 0$$

$2x - 3 = 0$ $\quad$ $x + 4 = 0$ $\quad$ $x + 2 = 0$

$x = \frac{3}{2}$ $\quad$ $x = -4$ $\quad$ $x = -2$

	$(-\infty,-4)$	$(-4,-2)$	$\left(-2,\frac{3}{2}\right)$	$\left(\frac{3}{2},\infty\right)$
$2x-3$	-	-	-	+
$x+4$	-	+	+	+
$x+2$	-	-	+	+
$\frac{(2x-3)(x+4)}{x+2}$	-	+	-	+

$\therefore The$ solution set $\left\{x: x<-4 \cup -2<x<\frac{3}{2}\right\}$

8. The first four terms of a binomial expansion $(1+ax)^n$ is

$$1+x-\frac{1}{2}x^2+px^3+\cdots$$

Find

(a) the values of a and n where $n \neq 0$.

(b) the value of p Hence, by substituting $x=\frac{1}{4}$, show that $\sqrt{\frac{3}{2}}$ is approximately equal to $\frac{157}{128}$.

SOLUTION

8(a)

Given that

$$(1+ax)^n = 1+x-\frac{1}{2}x^2+px^3+\cdots$$

From binomial expansion

$$(1+ax)^n = 1+(n)(ax)^1+\left(\frac{n(n-1)}{2!}\right)(ax)^2+\left(\frac{n(n-1)(n-2)}{3!}\right)(ax)^3+\cdots$$

$$(1+ax)^n = 1+nax+\frac{n(n-1)}{2}a^2x^2+\frac{n(n-1)(n-2)}{6}a^3x^3+\cdots$$

$$1+x-\frac{1}{2}x^2+px^3+\cdots = 1+nax+\frac{n(n-1)}{2}a^2x^2+\frac{n(n-1)(n-2)}{6}a^3x^3+\cdots$$

Coefficient of x:

$na = 1$ .... (1)

Coefficient of x^2:

$\frac{n(n-1)}{2}a^2 = -\frac{1}{2}$

$n(n-1)a^2 = -1$

$naa(n-1) = -1$

$a(n-1) = -1$ ....(2)

$na = 1$

$\frac{(2)}{(1)}$ $\quad \frac{a(n-1)}{na} = \frac{-1}{1}$

$$\frac{(n-1)}{n} = -1$$

$$n-1 = -n$$

$$2n = 1$$

$$n = \frac{1}{2}$$

From (1)

$$na = 1$$

$$\left(\frac{1}{2}\right)a = 1$$

$$a = 2$$

8(b)

Compare coefficient of x^3:

$$\frac{n(n-1)(n-2)}{6}a^3 = p$$

$$\frac{\left(\frac{1}{2}\right)\left[\left(\frac{1}{2}\right)-1\right]\left[\left(\frac{1}{2}\right)-2\right]}{6}(2)^3 = p$$

$$\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(8)}{6} = p$$

$$\frac{24}{8} = 6p$$

$$6p = 3$$

$$p = \frac{1}{2}$$

$$(1+2x)^{\frac{1}{2}} = 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \cdots$$

When $x = \frac{1}{4}$

$$\left[1 + 2\left(\frac{1}{4}\right)\right]^{\frac{1}{2}} = 1 + \frac{1}{4} - \frac{1}{2}\left(\frac{1}{4}\right)^2 + \frac{1}{2}\left(\frac{1}{4}\right)^3 + \cdots$$

$$\left[1 + \frac{1}{2}\right]^{\frac{1}{2}} = 1 + \frac{1}{4} - \frac{1}{2}\left(\frac{1}{16}\right) + \frac{1}{2}\left(\frac{1}{64}\right) + \cdots$$

$$\left[\frac{3}{2}\right]^{\frac{1}{2}} \approx 1 + \frac{1}{4} - \frac{1}{32} + \frac{1}{128}$$

$$\left[\frac{3}{2}\right]^{\frac{1}{2}} \approx \frac{1(128) + 1(32) - 1(4) + 1}{128}$$

$$\left[\frac{3}{2}\right]^{\frac{1}{2}} \approx \frac{157}{128}$$

$$\sqrt{\frac{3}{2}} \approx \frac{157}{128}$$

9. Given $f(x) = \ln(2x+3)$ and $g(x) = \frac{e^x - 3}{2}$.

(a) Show that $f(x)$ is a one-to-one function algebraically.

(b) Find $(f \circ g)(x)$ and $(g \circ f)(x)$. Hence, state the conclusion about the results.

(c) Sketch the graphs of $f(x)$ and $g(x)$ on the same axes. Hence, state the domain and range of $f(x)$.

SOLUTION

9(a)

$f(x) = \ln(2x+3)$

$f(x_1) = \ln(2x_1+3)$

$f(x_2) = \ln(2x_2+3)$

Let $f(x_1) = f(x_2)$

$\ln(2x_1+3) = \ln(2x_2+3)$

$2x_1 + 3 = 2x_2 + 3$

$2x_1 = 2x_2$

$x_1 = x_2$

Since $x_1 = x_2$, therefore $f(x)$ is a one-to-one function.

9(b)

$f(x) = \ln(2x+3)$, $\quad g(x) = \frac{e^x - 3}{2}$

$(f \circ g)(x) = f[g(x)]$

$= f\left[\frac{e^x - 3}{2}\right]$

$= \ln\left[2\left(\frac{e^x - 3}{2}\right) + 3\right]$

$= \ln[e^x - 3 + 3]$

$= \ln[e^x]$

$= x \ln e$

$= x$

$(g \circ f)(x) = g[f(x)]$

$= g[ln(2x+3)]$

$= \dfrac{e^{\ln(2x+3)} - 3}{2}$

$e^{\ln x} = x$

$e^{\ln(2x+3)} = 2x + 3$

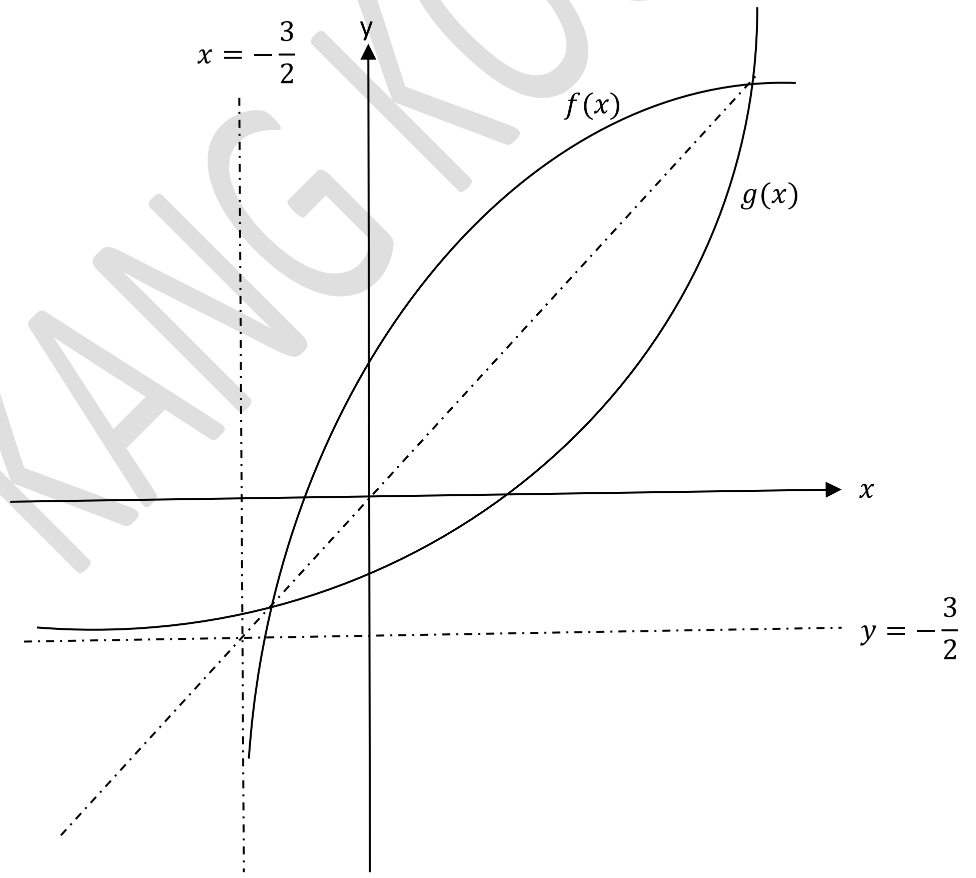
$= \dfrac{(2x+3) - 3}{2}$

$= x$

Conclusion

Since $f \circ g(x) = g \circ f(x) = x$, *therefore* $f^{-1}(x) = g(x)$ *and* $g^{-1}(x) = f(x)$.

9(c)



$$D_{f} = \left(-\frac{3}{2}, \infty\right) \qquad R_{f} = (-\infty, \infty)$$

10. Given

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 5 & 4 \\ 3 & 1 & 4 \end{bmatrix}$$

(a) Find the determinant of matrix A.

(b) Find the minor, cofactor and adjoint of matrix A.

(c) Given $A(adjoint(A)) = |A|I$ where I is 3 x 3 identity matrix, show that $A^{-1} = \frac{1}{|A|} adjoint(A)$. Hence, find A^{-1}.

(d) By using A^{-1} in part (c), solve the following simultaneous equations.

$$2x + 2y + 3z = 49$$
$$x + 5y + 4z = 74$$
$$3x + y + 4z = 49$$

SOLUTION

10(a)

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 5 & 4 \\ 3 & 1 & 4 \end{bmatrix}$$

$$|A| = (2)\begin{vmatrix} 5 & 4 \\ 1 & 4 \end{vmatrix} - (2)\begin{vmatrix} 1 & 4 \\ 3 & 4 \end{vmatrix} + (3)\begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix}$$

$$= (2)(20 - 4) - (2)(4 - 12) + (3)(1 - 15)$$

$$= (2)(16) - (2)(-8) + (3)(-14)$$

$$= 32 + 16 - 42$$

$$= 6$$

10(b)

$$Minor, M_{ij} = \begin{bmatrix} \begin{vmatrix} 5 & 4 \\ 1 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 5 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 1 & 5 \end{vmatrix} \end{bmatrix}$$
$$= \begin{bmatrix} (20 - 4) & (4 - 12) & (1 - 15) \\ (8 - 3) & (8 - 9) & (2 - 6) \\ (8 - 15) & (8 - 3) & (10 - 2) \end{bmatrix}$$

$$= \begin{bmatrix} 16 & -8 & -14 \\ 5 & -1 & -4 \\ -7 & 5 & 8 \end{bmatrix}$$

$$Cofactor, C_{ij} = \begin{bmatrix} +M_{11} & -M_{12} & +M_{13} \\ -M_{21} & +M_{22} & -M_{23} \\ +M_{31} & -M_{32} & +M_{33} \end{bmatrix}$$

$$= \begin{bmatrix} +(16) & -(-8) & +(-14) \\ -(5) & +(-1) & -(-4) \\ +(-7) & -(5) & +(8) \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 8 & -14 \\ -5 & -1 & 4 \\ -7 & -5 & 8 \end{bmatrix}$$

$$Adjoint\ A,\quad Adj\ A = C^T$$

$$= \begin{bmatrix} 16 & 8 & -14 \\ -5 & -1 & 4 \\ -7 & -5 & 8 \end{bmatrix}^T$$

$$= \begin{bmatrix} 16 & -5 & -7 \\ 8 & -1 & -5 \\ -14 & 4 & 8 \end{bmatrix}$$

10(c)

$$A(adjoint(A)) = |A|I$$

$$A^{-1}A = I$$

$$A^{-1}A(adjoint(A)) = A^{-1}|A|I$$

$$I(adjoint(A)) = A^{-1}|A|I$$

$$(adjoint(A)) = A^{-1}|A|$$

$$A^{-1}|A| = (adjoint(A))$$

$$A^{-1} = \frac{1}{|A|}(adjoint(A))$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 16 & -5 & -7 \\ 8 & -1 & -5 \\ -14 & 4 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{16}{6} & \frac{-5}{6} & \frac{-7}{6} \\ \frac{8}{6} & \frac{-1}{6} & \frac{-5}{6} \\ \frac{-14}{6} & \frac{4}{6} & \frac{8}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{8}{3} & \frac{-5}{6} & \frac{-7}{6} \\ \frac{4}{3} & \frac{-1}{6} & \frac{-5}{6} \\ \frac{-7}{3} & \frac{2}{3} & \frac{4}{3} \end{bmatrix}$$

10(d)

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

$$2x + 2y + 3z = 49$$
$$x + 5y + 4z = 74$$
$$3x + y + 4z = 49$$

$$\begin{bmatrix} 2 & 2 & 3 \\ 1 & 5 & 4 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 49 \\ 74 \\ 49 \end{bmatrix}$$

$$\begin{bmatrix} \frac{8}{3} & \frac{-5}{6} & \frac{-7}{6} \\ \frac{4}{3} & \frac{-1}{6} & \frac{-5}{6} \\ \frac{-7}{3} & \frac{2}{3} & \frac{4}{3} \end{bmatrix} \begin{bmatrix} 2 & 2 & 3 \\ 1 & 5 & 4 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{8}{3} & \frac{-5}{6} & \frac{-7}{6} \\ \frac{4}{3} & \frac{-1}{6} & \frac{-5}{6} \\ \frac{-7}{3} & \frac{2}{3} & \frac{4}{3} \end{bmatrix} \begin{bmatrix} 49 \\ 74 \\ 49 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{8}{3} & \frac{-5}{6} & \frac{-7}{6} \\ \frac{4}{3} & \frac{-1}{6} & \frac{-5}{6} \\ \frac{-7}{3} & \frac{2}{3} & \frac{4}{3} \end{bmatrix} \begin{bmatrix} 49 \\ 74 \\ 49 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{71}{6} \\ \frac{73}{6} \\ \frac{1}{3} \end{bmatrix}$$

$$\therefore \boldsymbol{x = \frac{71}{6}}, \quad \boldsymbol{y = \frac{73}{6}}, \quad \boldsymbol{z = \frac{1}{3}}$$