KEMENTERIAN PENDIDIKAN MALAYSIA

QS 015/1

Matriculation Programme Examination

Semester I

Session 2013/2014

1. Given matrices $A = \begin{bmatrix} 5 & 3 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} c & 3 & 0 \\ 3 & d & 0 \\ 0 & 0 & e \end{bmatrix}$. Find the values of c, d and e such that AB = 14I, where I is the identity matrix. Hence, determine A^{-1}.

2. Consider the function $f(x) = 1 + \ln(x)\,, x \geq 1$. Determine $f^{-1}(x)$ and state its range. Hence, evaluate $f^{-1}(3)$.

3. Find the value of x which satisfies the equation

   $log_9 x = (log_3 x)^2, \quad x > 1.$

4. Solve the equation $2^{2x-2} - 2^{x+1} = 2^x - 2^3$.

5. Given $g(x) = \frac{kx+8}{4x-5}, x \neq \frac{5}{4}$ where k is a constant.
   a. Find the value of k if $(g \circ g)(x) = x$.
   b. Find the value of k so that $g(x)$ is not a one-to-one function.

6. Given $f(x) = e^{3x} + 4, \ x \in \Re$.
   a. Find $f^{-1}(x)$.
   b. On the same axes, sketch the graphs of $f(x)$ and $f^{-1}(x)$. State the domain of $f(x)$ *and* $f^{-1}(x)$.

7. a. Find the values of p and q if $\frac{p}{4-2i} + \frac{q}{4+2i} = 1 + \frac{5}{2}i$.

   b. Given $log_{10}2 = m$ and $log_{10}7 = n$. Express x in terms of m and n if $(14^{3x+1})(8^{2x+3}) = 7$.

8. An osteoprorosis patient was advised by a doctor to take enough magnesium, vitamin D and calcium to improve bone density. In a week, the patient has to take 8 units magnesium, 11 units vitamin D and 17 units calcium. The following are three types of capsule that contains the three essential nutrients for the bone:

   Capsule of type P: 2 units magnesium, 1 unit vitamin D and 1 unit calcium.

   Capsule of type Q: 1 unit magnesium, 2 units vitamin D and 3 units calcium.

   Capsule of type R: 4 units magnesium, 6 units vitamin D and 10 units calcium.

   Let x, y and z represent the number of capsule of types P, Q and R respectively that the patient has to take in a week.

   (a) Obtain a system of linear equation to represent the given information and write the system in the form of matrix equation AX = B, where $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

   (b) Find the inverse of matrix A from part (a) by using the adjoint method. Hence, find the values of x, y and z.

   (c) The cost for each capsule of type P, Q and R are RM10, RM15 and RM17 respectively. How much will the expenses be for 4 weeks if the patient follws the doctor's advice?

9. a. In an arithmetic progression, the sum of the first four terms is 46 and the seventh term exceeds twice of the second term by 5. Obtain the first term and the common difference for the progression. Hence, calculate the sum of the first ten even terms of the progression.

   b. A ball is dropped from a height of 2m. Each time the ball hits the floor, it bounces vertically to a height that is $\frac{3}{4}$ of its previous height.

      i. Find the height of the ball at the tenth bounce.
      ii. Find the total distance that the ball will travel before the eleventh bounce.

10 a. Find the solution set of $|2 - 3x| < |x + 3|$.

   b. If x + 1 < 0, show that

      i. 2x – 1 < 0
      ii. $\frac{2x-1}{x+1} > 2$

1. Given matrices $A = \begin{bmatrix} 5 & 3 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} c & 3 & 0 \\ 3 & d & 0 \\ 0 & 0 & e \end{bmatrix}$. Find the values of c, d and e such that $AB = 14I$, where I is the identity matrix. Hence, determine A^{-1}.

SOLUTION

$$AB = 14I$$

$$\begin{bmatrix} 5 & 3 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} c & 3 & 0 \\ 3 & d & 0 \\ 0 & 0 & e \end{bmatrix} = \begin{bmatrix} 14 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 5c+9 & 15+3d & 0 \\ 3c-3 & 9-d & 0 \\ 0 & 0 & 2e \end{bmatrix} = \begin{bmatrix} 14 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 14 \end{bmatrix}$$

$5c + 9 = 14$ ➔ $c = 1$

$15 + 3d = 0$ ➔ $d = -5$

$2e = 14$ ➔ $e = 7$

$\therefore c = 1, d = -5, e = 7$

$$AB = 14I$$

$$A^{-1} = \frac{1}{14} B$$

$$A^{-1} = \frac{1}{14}\begin{bmatrix} 1 & 3 & 0 \\ 3 & -5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{14} & \frac{3}{14} & 0 \\ \frac{3}{14} & \frac{-5}{14} & 0 \\ 0 & 0 & \frac{7}{14} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{14} & \frac{3}{14} & 0 \\ \frac{3}{14} & \frac{-5}{14} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

2. Consider the function $f(x) = 1 + \ln(x), x \geq 1$. Determine $f^{-1}(x)$ and state its range. Hence, evaluate $f^{-1}(3)$.

SOLUTION

Method I	***Method II***
$y = \ln x + 1, \quad x \geq 1$ $\ln x = y - 1$ $x = e^{y-1}$ $f^{-1}(x) = e^{x-1}$ $R_{f^{-1}} = \{y: y \geq 1\}$ $f^{-1}(3) = e^{3-1} = e^2$	$f(x) = 1 + \ln(x)$ $f(f^{-1}(x)) = x$ $1 + \ln(f^{-1}(x)) = x$ $\ln(f^{-1}(x)) = x - 1$ $f^{-1}(x) = e^{x-1}$ $R_{f^{-1}} = \{y: y \geq 1\}$ $f^{-1}(3) = e^{3-1} = e^2$

3. Find the value of x which satisfies the equation

$$log_9 x = (log_3 x)^2, \quad x > 1.$$

SOLUTION

$$log_9 x = (log_3 x)^2$$

$$\frac{log_3 x}{log_3 9} = (log_3 x)^2$$

$$\frac{log_3 x}{log_3 3^2} = (log_3 x)^2$$

$$\frac{log_3 x}{2 log_3 3} = (log_3 x)^2$$

$$\frac{log_3 x}{2} = (log_3 x)^2$$

$$log_3 x = 2(log_3 x)^2$$

$$Let\ u = log_3 x$$

$$u = 2u^2$$

$$2u^2 - u = 0$$

$$u(2u - 1) = 0$$

$u = 0$ or $(2u - 1) = 0$

$u = 0$ or $u = \frac{1}{2}$

$log_3 x = 0$ or $log_3 x = \frac{1}{2}$

$x = 3^0$ or $x = 3^{\frac{1}{2}}$

$x = 1$ or $x = \sqrt{3}$

$$Since\ x > 1, the\ solution\ is\ x = \sqrt{3}$$

4. Solve the equation $2^{2x-2} - 2^{x+1} = 2^x - 2^3$.

SOLUTION

$$2^{2x-2} - 2^{x+1} = 2^x - 2^3$$

$$2^{2x}2^{-2} - 2^x 2^1 = 2^x - 2^3$$

$$\frac{(2^x)^2}{4} - 2(2^x) = 2^x - 8$$

$$Let\ u = 2^x$$

$$\frac{(u)^2}{4} - 2(u) = u - 8$$

$$u^2 - 8u = 4u - 32$$

$$u^2 - 8u - 4u + 32 = 0$$

$$u^2 - 12u + 32 = 0$$

$$(u - 8)(u - 4) = 0$$

$u - 8 = 0$ or $u - 4 = 0$

$u = 8$ or $u = 4$

$2^x = 8$ or $2^x = 4$

$x = 3$ or $x = 2$

5. Given $g(x) = \frac{kx+8}{4x-5}$, $x \neq \frac{5}{4}$ where k is a constant.
   a. Find the value of k if $(g \circ g)(x) = x$.
   b. Find the value of k so that $g(x)$ is not a one-to-one function.

SOLUTION

5a.

$$g(x) = \frac{kx + 8}{4x - 5}$$

$$(g \circ g)(x) = x$$

$$g[g(x)] = x$$

$$\frac{k\left(\frac{kx+8}{4x-5}\right) + 8}{4\left(\frac{kx+8}{4x-5}\right) - 5} = x$$

$$\frac{k\left(\frac{kx+8}{4x-5}\right) + \frac{8(4x-5)}{(4x-5)}}{4\left(\frac{kx+8}{4x-5}\right) - \frac{5(4x-5)}{(4x-5)}} = x$$

$$\frac{\frac{k(kx+8) + 8(4x-5)}{4x-5}}{\frac{4(kx+8) - 5(4x-5)}{4x-5}} = x$$

$$\frac{k(kx+8) + 8(4x-5)}{4(kx+8) - 5(4x-5)} = x$$

$$(k^2x + 8k) + (32x - 40) = x[(4kx + 32) - (20x - 25)]$$

$$k^2x + 8k + 32x - 40 = x[4kx + 32 - 20x + 25]$$

$$(k^2 + 32)x + (8k - 40) = 4kx^2 + 32x - 20x^2 + 25x$$

$$(k^2 + 32)x + (8k - 40) = (4k - 20)x^2 + 57x$$

Compare the coeeficient of x^2

$$4k - 20 = 0$$

$$k = 5$$

5b.

$$g(x) = \frac{kx+8}{4x-5}$$

Let $x_2 \neq x_1$, but $g(x_2) = g(x_1)$

$$\frac{kx_2+8}{4x_2-5} = \frac{kx_1+8}{4x_1-5}$$

$$(kx_2+8)(4x_1-5) = (kx_1+8)(4x_2-5)$$

$$4kx_1x_2 - 5kx_2 + 32x_1 - 40 = 4kx_1x_2 - 5kx_1 + 32x_2 - 40$$

$$32x_1 - 5kx_2 - 32x_2 + 5kx_1 = 0$$

$$32x_1 - 32x_2 + 5kx_1 - 5kx_2 = 0$$

$$32(x_1 - x_2) + 5k(x_1 - x_2) = 0$$

$$(x_1 - x_2)(32 + 5k) = 0$$

$x_1 - x_2 \neq 0$ ➔ $32 + 5k = 0$

$$k = \frac{-32}{5}$$

6. Given $f(x) = e^{3x} + 4,\ x \in \Re$.
   a. Find $f^{-1}(x)$.
   b. On the same axes, sketch the graphs of $f(x)$ and $f^{-1}(x)$. State the domain of $f(x)$ *and* $f^{-1}(x)$.

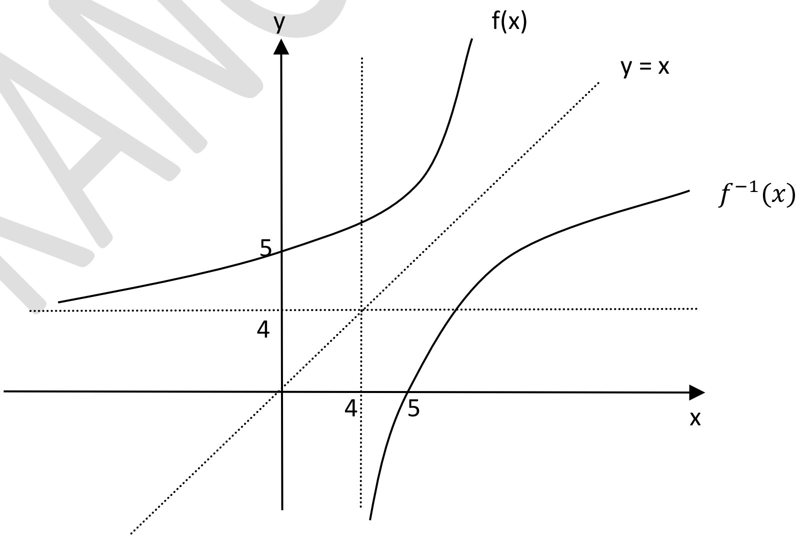
SOLUTION

6a

$$f(x) = e^{3x} + 4$$

Method I	Method II
Let $y = e^{3x} + 4$ $e^{3x} = y - 4$ $\ln(e^{3x}) = \ln(y-4)$ $3x \ln e = \ln(y-4)$ $3x = \ln(y-4)$ $x = \frac{1}{3}\ln(y-4)$ $\therefore f^{-1}(x) = \frac{1}{3}\ln(x-4)$	$f[f^{-1}(x)] = x$ $e^{3f^{-1}(x)} + 4 = x$ $e^{3f^{-1}(x)} = x - 4$ $\ln e^{3f^{-1}(x)} = ln(x-4)$ $3f^{-1}(x) \ln e = ln(x-4)$ $3f^{-1}(x) = ln(x-4)$ $f^{-1}(x) = \frac{1}{3}\ln(x-4)$

6b



Domain $f(x)$: $\{-\infty, \infty\}$ Domain $f^{-1}(x)$: $\{4, \infty\}$

7. a. Find the values of p and q if $\frac{p}{4-2i}+\frac{q}{4+2i}=1+\frac{5}{2}i$.

b. Given $log_{10}2=m$ and $log_{10}7=n$. Express x in terms of m and n if $(14^{3x+1})(8^{2x+3})=7$.

SOLUTION

7a

$$\frac{p}{4-2i}+\frac{q}{4+2i}=1+\frac{5}{2}i$$

$$\frac{p(4+2i)+q(4-2i)}{(4-2i)(4+2i)}=\frac{2+5i}{2}$$

$$\frac{4p+2pi+4q-2qi}{16+8i-8i-4i^2}=\frac{2+5i}{2}$$

$$\frac{4p+4q+2pi-2qi}{16-4(-1)}=\frac{2+5i}{2}$$

$$\frac{(4p+4q)+(2p-2q)i}{20}=\frac{2+5i}{2}$$

$$2[(4p+4q)+(2p-2q)i]=20[2+5i]$$

$$[(4p+4q)+(2p-2q)i]=\frac{20}{2}[2+5i]$$

$$[(4p+4q)+(2p-2q)i]=10[2+5i]$$

$$[(4p+4q)+(2p-2q)i]=[20+50i]$$

$(4p+4q)=20$

$p+q=5$ .... (1)

$2p-2q=50$

$p-q=25$........................ (2)

$(1)+(2)$

$2p=30$

$p=15$

$q=-10$

7b

$log_{10}2 = m$, $log_{10}7 = n$

$(14^{3x+1})(8^{2x+3}) = 7$

$[(2\ x\ 7)^{3x+1}][(2^3)^{2x+3}] = 7$

$[2^{3x+1}\ x\ 7^{3x+1}][(2)^{6x+9}] = 7$

$[2^{3x+1}\ x(2)^{6x+9}\ ][7^{3x+1}] = 7$

$[2^{3x+1+6x+9}\ ][7^{3x+1}] = 7$

$[2^{9x+10}\ ][7^{3x+1}] = 7$

$log\{[2^{9x+10}\ ][7^{3x+1}]\} = \log 7$

$log2^{9x+10} + log7^{3x+1} = \log 7$

$(9x + 10) \log 2 + (3x + 1) \log 7 = \log 7$

$(9x + 10)m + (3x + 1)n = n$

$9xm + 10m + 3xn + n = n$

$9xm + 3xn = n - n - 10m$

$x(9m + 3n) = -10m$

$$x = \frac{-10m}{9m + 3n}$$

8. An osteoprorosis patient was advised by a doctor to take enough magnesium, vitamin D and calcium to improve bone density. In a week, the patient has to take 8 units magnesium, 11 units vitamin D and 17 units calcium. The following are three types of capsule that contains the three essential nutrients for the bone:

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Let x, y and z represent the number of capsule of types P, Q and R respectively that the patient has to take in a week.

(a) Obtain a system of linear equation to represent the given information and write the system in the form of matrix equation AX = B, where $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

(b) Find the inverse of matrix A from part (a) by using the adjoint method. Hence, find the values of x, y and z.

(c) The cost for each capsule of type P, Q and R are RM10, RM15 and RM17 respectively. How much will the expenses be for 4 weeks if the patient follws the doctor's advice?

SOLUTION

8a

$$2x + y + 4z = 8$$

$$x + 2y + 6z = 11$$

$$x + 3y + 10z = 17$$

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & 2 & 6 \\ 1 & 3 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \\ 17 \end{bmatrix}$$

8b

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 2 & 6 \\ 1 & 3 & 10 \end{bmatrix}$$

$$|A| = (2)\begin{vmatrix} 2 & 6 \\ 3 & 10 \end{vmatrix} - (1)\begin{vmatrix} 1 & 6 \\ 1 & 10 \end{vmatrix} + (4)\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$

$$|A| = (2)[20 - 18] - (1)[10 - 6] + (4)[3 - 2]$$

$$|A| = (2)[2] - (1)[4] + (4)[1]$$

$$|A| = 4$$

$$Cofactor\ of\ A = \begin{bmatrix} +\begin{vmatrix} 2 & 6 \\ 3 & 10 \end{vmatrix} & -\begin{vmatrix} 1 & 6 \\ 1 & 10 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} \\ -\begin{vmatrix} 1 & 4 \\ 3 & 10 \end{vmatrix} & +\begin{vmatrix} 2 & 4 \\ 1 & 10 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} \\ +\begin{vmatrix} 1 & 4 \\ 2 & 6 \end{vmatrix} & -\begin{vmatrix} 2 & 4 \\ 1 & 6 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \end{bmatrix}$$

$$Cofactor\ of\ A = \begin{bmatrix} +(20-18) & -(10-6) & +(3-2) \\ -(10-12) & +(20-4) & -(6-1) \\ +(6-8) & -(12-4) & +(4-1) \end{bmatrix}$$

$$Cofactor\ of\ A = \begin{bmatrix} 2 & -4 & 1 \\ 2 & 16 & -5 \\ -2 & -8 & 3 \end{bmatrix}$$

$$Adjoin\ of\ A = C^T$$

$$Adjoin\ of\ A = \begin{bmatrix} 2 & 2 & -2 \\ -4 & 16 & -8 \\ 1 & -5 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} Adj\ A$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & 2 & -2 \\ -4 & 16 & -8 \\ 1 & -5 & 3 \end{bmatrix}$$

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B$$

$$X = \frac{1}{4}\begin{bmatrix} 2 & 2 & -2 \\ -4 & 16 & -8 \\ 1 & -5 & 3 \end{bmatrix}\begin{bmatrix} 8 \\ 11 \\ 17 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore x = 1, \qquad y = 2, \qquad z = 1$$

8c

$$P = 10, \qquad Q = 15, \qquad R = 17$$

$$Expenses\ for\ 4\ weeks = 4[10(1) + 15(2) + 17(1)]$$

$$Expenses\ for\ 4\ weeks = 4[57]$$

$Expenses\ for\ 4\ weeks\ =\ RM228$

9. a. In an arithmetic progression, the sum of the first four terms is 46 and the seventh term exceeds twice of the second term by 5. Obtain the first term and the common difference for the progression. Hence, calculate the sum of the first ten even terms of the progression.

   b. A ball is dropped from a height of 2m. Each time the ball hits the floor, it bounces vertically to a height that is $\frac{3}{4}$ of its previous height.

      i. Find the height of the ball at the tenth bounce.
      ii. Find the total distance that the ball will travel before the eleventh bounce.

SOLUTION

$9a$

Arithmetic progression $S_n = \frac{n}{2}[2a + (n-1)d],\ T_n = a + (n-1)d$

$S_4 = 46$

$\frac{4}{2}[2a + (4-1)d] = 46$

$2[2a + 3d] = 46$

$4a + 6d = 46$

$2a + 3d = 23$ .... (1)

$T_7 = 2T_2 + 5$

$a + (7-1)d = 2[a + (2-1)d] + 5$

$a + 6d = 2[a + d] + 5$

$a + 6d = 2a + 2d + 5$

$a - 4d = -5$

$2a - 8d = -10$ ....(2)

$(1) - (2)$

$11d = 33$

$d = 3$

$a = 7$

9b(i)

1st bounce ➔ $T_1 = 2\left(\frac{3}{4}\right)$

2nd bounce ➔ $T_2 = 2\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = 2\left(\frac{3}{4}\right)^2$

3rd bounce ➔ $T_3 = 2\left(\frac{3}{4}\right)^3$

$a = 2\left(\frac{3}{4}\right), \qquad r = \left(\frac{3}{4}\right)$

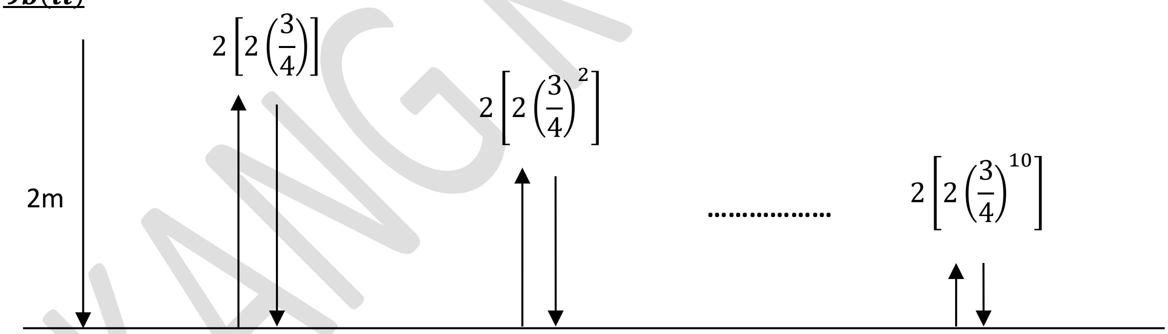
$$T_r = ar^{n-1}$$

$$T_{10} = \left[2\left(\frac{3}{4}\right)\right]\left[\left(\frac{3}{4}\right)^{10-1}\right]$$

$$T_{10} = \left[2\left(\frac{3}{4}\right)\right]\left[\left(\frac{3}{4}\right)^{9}\right]$$

$$T_{10} = 2\left(\frac{3}{4}\right)^{10}$$

9b(ii)



$$Total\ distance = 2 + 2\left[2\left(\frac{3}{4}\right)\right] + 2\left[2\left(\frac{3}{4}\right)^2\right] + \ldots + 2\left[2\left(\frac{3}{4}\right)^{10}\right]$$

$$= 2\left[1 + 2\left(\frac{3}{4}\right) + 2\left(\frac{3}{4}\right)^2 + \cdots + 2\left(\frac{3}{4}\right)^{10}\right]$$

$$= 2\left\{1 + 2\left[\underbrace{\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 + \cdots + \left(\frac{3}{4}\right)^{10}}\right]\right\}$$

$\boldsymbol{a = \left(\frac{3}{4}\right), \qquad r = \left(\frac{3}{4}\right)}$

$\boldsymbol{S_n = \frac{a(1 - r^n)}{1 - r}}$

$$= 2\left\{1 + 2\left[\frac{\left(\frac{3}{4}\right)\left[1 - \left(\frac{3}{4}\right)^{10}\right]}{1 - \left(\frac{3}{4}\right)}\right]\right\}$$

$$= 13.324$$

10 a. Find the solution set of $|2 - 3x| < |x + 3|$.

b. If x + 1 < 0, show that

i. 2x – 1 < 0

ii. $\frac{2x-1}{x+1} > 2$

SOLUTION

$\mathbf{10a}$

$$|2 - 3x| < |x + 3|$$

$$(2 - 3x)^2 < (x + 3)^2$$

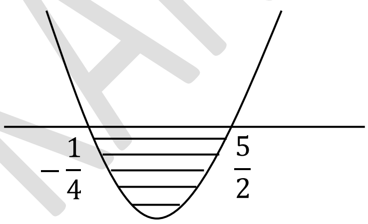
$$4 - 12x + 9x^2 < x^2 + 6x + 9$$

$$9x^2 - x^2 - 12x - 6x + 4 - 9 < 0$$

$$8x^2 - 18x - 5 < 0$$

$$(2x - 5)(4x + 1) < 0$$

$$x = \frac{5}{2}, -\frac{1}{4}$$



$$\therefore \textit{Solution set is } \left\{x: -\frac{1}{4} < x < \frac{5}{2}\right\}$$

$\mathbf{10b(i)}$

$$2x - 1 = 2x + 2 - 3 = 2(x + 1) - 3$$

$$\because x + 1 < 0 \rightarrow 2(x + 1) - 3 < 0$$

$\therefore 2x - 1 < 0$

10b(ii)

$$\frac{2x-1}{x+1} > 2$$

$$\frac{2x-1}{x+1} - 2 > 0$$

$$\frac{2x-1}{x+1} - 2 = \frac{(2x-1)-2(x+1)}{x+1}$$

$$= \frac{2x-1-2x-2}{x+1}$$

$$= \frac{-3}{x+1}$$

$$\because x+1 < 0$$

$$\frac{-3}{x+1} > 0$$

$$\frac{2x-1}{x+1} - 2 > 0$$

$$\therefore \frac{2x-1}{x+1} > 2$$