

QS 015/1
Matriculation Programme
Examination
Semester I
Session 2014/2015

1. Solve the equation $3^x + 3^{(3-x)} = 12$.
2. Solve the inequality $\frac{1}{6-x} < \frac{1}{x-1}$.
3. Given matrices $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ a & b & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ z & 1 & 0 \\ x & y & 1 \end{bmatrix}$ where B is the inverse of A . Find x, y and z in terms of a and b .
4. Using algebraic method, find the least value of n for which the sum of the first n terms of a geometric series

$$0.88 + (0.88)^2 + (0.88)^3 + (0.88)^4 + \dots$$

is greater than half of its sum to infinity.

5. (a) State the interval for x such that the expansion for $(4 + 3x)^{\frac{3}{2}}$ is valid.
(b) Expand $(4 + 3x)^{\frac{3}{2}}$ in ascending power of x up to the term in x^3 .
(c) Hence, by substituting an appropriate value of x , evaluate $(5)^{\frac{3}{2}}$ correct to three decimal places.
6. (a) Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$.
 - (i) Find $(f - g)(x)$.
 - (ii) Evaluate $(3g - 2)(1)$.

(b) Given $f(x) = \sqrt{2x + \frac{1}{2}}$. State the domain and range of $f(x)$. Hence, on the same axes, sketch the graph of $f(x)$ and $f^{-1}(x)$.

7. Let $z = a + bi$ be a nonzero complex number.

(a) Show that $\frac{1}{z} = \frac{\bar{z}}{|z|^2}$.

(b) Show that if $\bar{z} = -z$, then z is a complex number with only an imaginary part.

(c) Find the value of a and b if $z(2 - i) = (\bar{z} + 1)(1 + i)$.

8. (a) Solve the for the following equation $|6x^2 + x - 11| = 4$.
- (b) Find the solution set for the inequality

$$2 - \left(\frac{x+2}{x-4}\right) < 5$$

9. Two companies P and Q decided to award prizes to their employees for three work ethical values, namely punctuality (x), creativity (y) and efficiency (x). Company P decided to award a total of RM3850 for the three values to 6, 2 and 3 employees respectively, while company Q decided to award RM3200 for the three values to 4, 1 and 5 employees respectively. The total amount for all the three prizes is RM1000.
- (a) Construct a system of linear equations to represent the above situation.
- (b) By forming a matrix equation, solve this equation system using the elimination method.
- (c) With the same total amount of money spent by company P and Q, is it possible for company P to award 15 employees for their creativity instead of 2 employees? Give your reason.
10. (a) Determine wheather $f(x) = \frac{1}{x-4}$ and $g(x) = \frac{4x+1}{x}$ are inverse function of each other by computing their composite functions.
- (b) Given $f(x) = \ln(1 - 3x)$.
- (i) Determine the domain and range of $f(x)$. Then sketch the graph of $f(x)$.
- (ii) Find $f^{-1}(x)$, if it exists. Hence, state the domain and range of $f^{-1}(x)$.

END OF QUESTION PAPER

1. Solve the equation $3^x + 3^{(3-x)} = 12$.

SOLUTION

$$3^x + 3^{(3-x)} = 12$$

$$3^x + 3^3 \cdot 3^{-x} = 12$$

$$3^x + 3^3 \cdot \frac{1}{3^x} = 12$$

$$3^x + \frac{27}{3^x} = 12$$

Let $y = 3^x$

$$y + \frac{27}{y} = 12$$

$$\frac{y^2 + 27}{y} = 12$$

$$y^2 + 27 = 12y$$

$$y^2 - 12y + 27 = 0$$

$$(y - 9)(y - 3) = 0$$

$$(y - 9) = 0$$

$$y = 9$$

$$3^x = 9$$

$$x = 2$$

$$(y - 3) = 0$$

$$y = 3$$

$$3^x = 3$$

$$x = 1$$

2. Solve the inequality $\frac{1}{6-x} < \frac{1}{x-1}$.

SOLUTION

$$\frac{1}{6-x} < \frac{1}{x-1}$$

$$\frac{1}{6-x} - \frac{1}{x-1} < 0$$

$$\frac{(x-1) - (6-x)}{(6-x)(x-1)} < 0$$



$$\frac{x-1-6+x}{(6-x)(x-1)} < 0$$

$$\frac{2x-7}{(6-x)(x-1)} < 0$$

$$\begin{aligned} 2x-7 &= 0 \\ x &= \frac{7}{2} \end{aligned}$$

$$\begin{aligned} 6-x &= 0 \\ x &= 6 \end{aligned}$$

$$\begin{aligned} x-1 &= 0 \\ x &= 1 \end{aligned}$$

	$(-\infty, 1)$	$(1, \frac{7}{2})$	$(\frac{7}{2}, 6)$	$(6, \infty)$
$2x-7$	-	-	+	+
$6-x$	+	+	+	-
$x-1$	-	+	+	+
$\frac{2x-7}{(6-x)(x-1)}$	+		+	

$$\therefore \left(1, \frac{7}{2}\right) \cup (6, \infty)$$

3. Given matrices $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ a & b & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ z & 1 & 0 \\ x & y & 1 \end{bmatrix}$ where B is the inverse of A . Find x, y and z in terms of a and b .

SOLUTION

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ a & b & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ z & 1 & 0 \\ x & y & 1 \end{bmatrix}$$

Since B is the inverse of $A \quad \rightarrow \quad AB = I$

$$AB = I$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ a & b & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ z & 1 & 0 \\ x & y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 4+z+0 & 0+1+0 & 0+0+0 \\ a+bz+x & 0+b+y & 0+0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4+z & 1 & 0 \\ a+bz+x & b+y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$4+z=0 \quad \rightarrow \quad z=-4$$

$$a+bz+x=0 \quad \rightarrow \quad x=-a+4b$$

$$b+y=0 \quad \rightarrow \quad y=-b$$

4. Using algebraic method, find the least value of n for which the sum of the first n terms of a geometric series

$$0.88 + (0.88)^2 + (0.88)^3 + (0.88)^4 + \dots$$

is greater than half of its sum to infinity.

SOLUTION

$$s_n > \frac{1}{2} s_\infty$$

$$a = 0.88, r = 0.88$$

$$\frac{a(1-r^n)}{1-r} > \frac{1}{2} \left(\frac{a}{1-r} \right)$$

$$\frac{(0.88)[1-(0.88)^n]}{1-0.88} > \frac{1}{2} \left(\frac{0.88}{1-0.88} \right)$$

$$[1-(0.88)^n] > \frac{1}{2} \left(\frac{0.88}{1-0.88} \right) \left(\frac{1-0.88}{0.88} \right)$$

$$1-(0.88)^n > \frac{1}{2}$$

$$1-\frac{1}{2} > (0.88)^n$$

$$\frac{1}{2} > (0.88)^n$$

$$(0.88)^n < \frac{1}{2}$$

$$\log (0.88)^n < \log \left(\frac{1}{2} \right)$$

$$n \log 0.88 < -0.3010$$

$$n(-0.0555) < -0.3010$$

$$-0.0555n < -0.3010$$

$$0.3010 < 0.0555n$$

$$0.0555n > 0.3010$$

Geometric Series

$$s_n = \frac{a(1-r^n)}{1-r}$$

$$s_\infty = \left(\frac{a}{1-r} \right)$$

$$n > \frac{0.3010}{0.0555}$$

$$n > 5.422$$

$$n = 6$$

5. (a) State the interval for x such that the expansion for $(4 + 3x)^{\frac{3}{2}}$ is valid.
- (b) Expand $(4 + 3x)^{\frac{3}{2}}$ in ascending power of x up to the term in x^3 .
- (c) Hence, by substituting an appropriate value of x , evaluate $(5)^{\frac{3}{2}}$ correct to three decimal places.

SOLUTION

$$\text{a) } (4 + 3x)^{\frac{3}{2}} = \left[4 \left(1 + \frac{3}{4}x\right)\right]^{\frac{3}{2}}$$

The expansion is valid for

$$-1 < \frac{3}{4}x < 1$$

$$-\frac{4}{3} < x < \frac{4}{3}$$

$$\begin{aligned} \text{b) } (4 + 3x)^{\frac{3}{2}} &= \left[4 \left(1 + \frac{3}{4}x\right)\right]^{\frac{3}{2}} \\ &= 4^{\frac{3}{2}} \left(1 + \frac{3}{4}x\right)^{\frac{3}{2}} \\ &= 8 \left(1 + \frac{3}{4}x\right)^{\frac{3}{2}} \\ &= 8 \left[1 + \frac{\binom{3}{2}}{1!} \left(\frac{3}{4}x\right) + \frac{\binom{3}{2}\binom{1}{2}}{2!} \left(\frac{3}{4}x\right)^2 + \frac{\binom{3}{2}\binom{1}{2}\binom{-1}{2}}{3!} \left(\frac{3}{4}x\right)^3 + \dots\right] \\ &= 8 \left[1 + \frac{9}{8}x + \frac{3}{8} \left(\frac{9}{16}x^2\right) - \frac{3}{48} \left(\frac{27}{64}x^3\right) + \dots\right] \\ &= 8 \left[1 + \frac{9}{8}x + \frac{27}{128}x^2 - \frac{81}{3072}x^3 + \dots\right] \\ &= 8 \left[1 + \frac{9}{8}x + \frac{27}{128}x^2 - \frac{27}{1024}x^3 + \dots\right] \\ &= 8 + 9x + \frac{27}{16}x^2 - \frac{27}{128}x^3 + \dots \end{aligned}$$

$$\text{c) } (4 + 3x)^{\frac{3}{2}} = 8 + 9x + \frac{27}{16}x^2 - \frac{27}{128}x^3 + \dots$$

$$\begin{aligned} (4 + 3x)^{\frac{3}{2}} &= (5)^{\frac{3}{2}} \\ 4 + 3x &= 5 \end{aligned}$$

$$3x = 1$$
$$x = \frac{1}{3}$$

$$(4 + 3x)^{\frac{3}{2}} = 8 + 9x + \frac{27}{16}x^2 - \frac{27}{128}x^3 + \dots$$

$$\left[4 + 3\left(\frac{1}{3}\right)\right]^{\frac{3}{2}} = 8 + 9\left(\frac{1}{3}\right) + \frac{27}{16}\left(\frac{1}{3}\right)^2 - \frac{27}{128}\left(\frac{1}{3}\right)^3 + \dots$$

$$[5]^{\frac{3}{2}} = 8 + 9\left(\frac{1}{3}\right) + \frac{27}{16}\left(\frac{1}{3}\right)^2 - \frac{27}{128}\left(\frac{1}{3}\right)^3 + \dots$$
$$= 11.180$$

6. (a) Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$.

(i) Find $(f - g)(x)$.

(ii) Evaluate $(3g - 2f)(1)$.

(b) Given $f(x) = \sqrt{2x + \frac{1}{2}}$. State the domain and range of $f(x)$. Hence, on the same axes, sketch the graph of $f(x)$ and $f^{-1}(x)$.

SOLUTION

a) $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$

$$\begin{aligned} \text{i. } (f - g)(x) &= f(x) - g(x) \\ &= (2x + 1) - (x^2 + 2x - 1) \\ &= 2x + 1 - x^2 - 2x + 1 \\ &= -x^2 + 2 \end{aligned}$$

$$\begin{aligned} \text{ii. } (3g - 2f)(x) &= 3g(x) - 2f(x) \\ &= 3(x^2 + 2x - 1) - 2(2x + 1) \\ &= 3x^2 + 6x - 3 - 4x - 2 \\ &= 3x^2 + 2x - 5 \end{aligned}$$

$$\begin{aligned} (3g - 2f)(1) &= 3(1)^2 + 2(1) - 5 \\ &= 3 + 2 - 5 \\ &= 0 \end{aligned}$$

$$\text{b) } f(x) = \sqrt{2x + \frac{1}{2}}$$

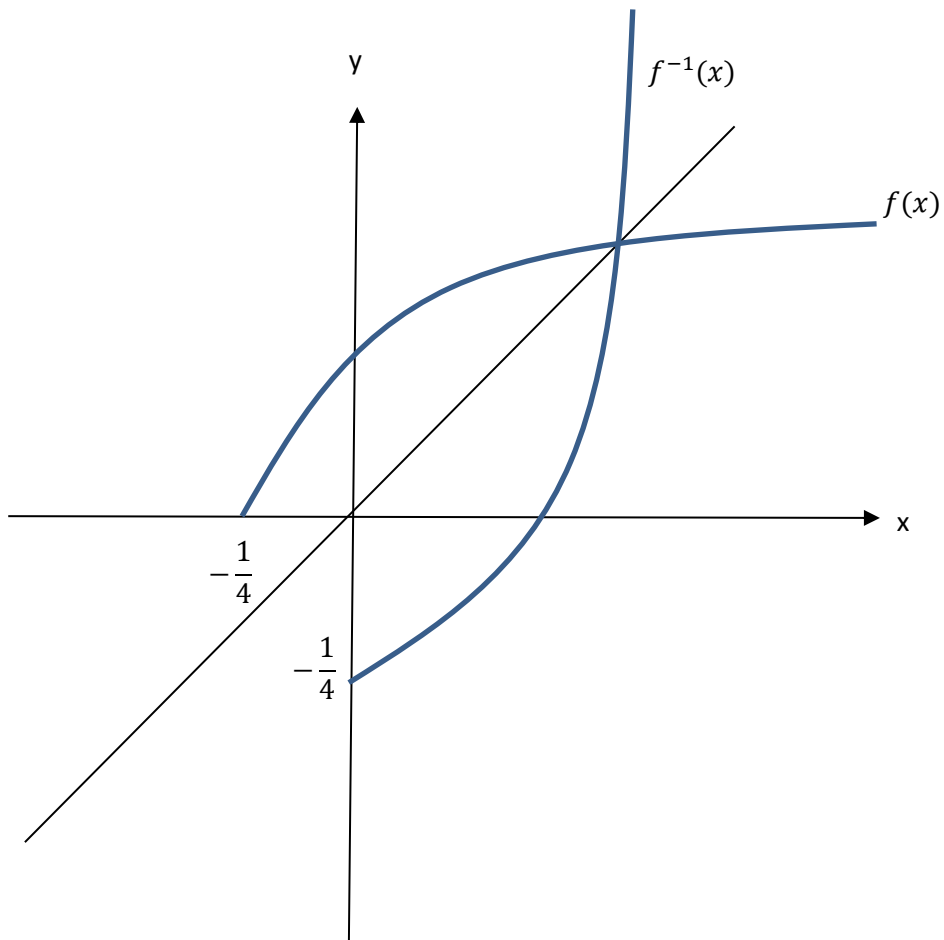
$$D_f: 2x + \frac{1}{2} \geq 0$$

$$2x \geq -\frac{1}{2}$$

$$x \geq -\frac{1}{4}$$

$$D_f = \left\{ x: x \geq -\frac{1}{4} \right\}$$

$$R_f = \{ y: y \geq 0 \}$$



7. Let $z = a + bi$ be a nonzero complex number.

- Show that $\frac{1}{z} = \frac{\bar{z}}{|z|^2}$.
- Show that if $\bar{z} = -z$, then z is a complex number with only an imaginary part.
- Find the value of a and b if $z(2 - i) = (\bar{z} + 1)(1 + i)$.

SOLUTION

a) $z = a + bi$

$$\begin{aligned}\frac{\bar{z}}{|z|^2} &= \frac{a-bi}{|a+bi|^2} \\ &= \frac{a-bi}{(\sqrt{a^2+b^2})^2} \\ &= \frac{a-bi}{a^2+b^2} \\ &= \frac{a-bi}{a^2+b^2} \left(\frac{a+bi}{a+bi} \right) \\ &= \frac{a^2+abi-abi-(bi)^2}{(a^2+b^2)(a+bi)} \\ &= \frac{a^2-b^2i^2}{(a^2+b^2)(a+bi)} \\ &= \frac{a^2-b^2(-1)}{(a^2+b^2)(a+bi)} \\ &= \frac{a^2+b^2}{(a^2+b^2)(a+bi)} \\ &= \frac{1}{a+bi} \\ &= \frac{1}{z}\end{aligned}$$

$$\text{b) } \bar{z} = -z$$

$$a - bi = -(a + bi)$$

$$a - bi = -a - bi$$

$$a = -a$$

$$2a = 0$$

$$a = 0$$

$$\text{Since } z = a + bi$$

$$= 0 + bi$$

$$= bi$$

$\therefore z$ is a complex number with only an imaginary part when $\bar{z} = -z$

$$\text{c) } z(2 - i) = (\bar{z} + 1)(1 + i)$$

$$(a + bi)(2 - i) = [(a - bi) + 1](1 + i)$$

$$2a + 2bi - ai - bi^2 = (a - bi)(1 + i) + (1 + i)$$

$$2a + 2bi - ai + b = (a - bi + ai - bi^2) + (1 + i)$$

$$2a + b + (2b - a)i = a - bi + ai + b + 1 + i$$

$$2a + (2b - a)i = (a + 1) + (a - b + 1)i$$

By equating real part

$$2a = a + 1$$

$$a = 1$$

By equating imaginary part

$$(2b - a) = (a - b + 1)$$

$$3b - 2a = 1$$

$$3b - 2(1) = 1$$

$$3b - 2 = 1$$

$$3b = 3$$

$$b = 1$$

8. (a) Solve the for the following equation $|6x^2 + x - 11| = 4$.
- (b) Find the solution set for the inequality

$$2 - \left(\frac{x+2}{x-4}\right) < 5$$

SOLUTION

$$|x| = a \Leftrightarrow x = a \text{ or } x = -a$$

a) $|6x^2 + x - 11| = 4$

$$6x^2 + x - 11 = 4$$

or

$$6x^2 + x - 11 = -4$$

$$6x^2 + x - 15 = 0$$

$$6x^2 + x - 7 = 0$$

$$(3x + 5)(2x - 3) = 0$$

$$(6x + 7)(x - 1) = 0$$

$$3x + 5 = 0 \quad 2x - 3 = 0$$

$$6x + 7 = 0 \quad x - 1 = 0$$

$$x = -\frac{5}{3}, \quad x = \frac{3}{2}$$

$$x = -\frac{7}{6}, \quad x = 1$$

$$\therefore x = -\frac{5}{3}, \quad x = \frac{3}{2}, \quad x = -\frac{7}{6}, \quad x = 1$$

b) $2 - \left(\frac{x+2}{x-4}\right) < 5$

$$2 - \left(\frac{x+2}{x-4}\right) - 5 < 0$$

$$\frac{2(x-4) - (x+2) - 5(x-4)}{x-4} < 0$$

$$\frac{2x - 8 - x - 2 - 5x + 20}{x-4} < 0$$

$$\frac{-4x + 10}{x-4} < 0$$

$$-4x + 10 = 0 \quad x - 4 = 0$$

$$x = \frac{10}{4} = \frac{5}{2} \quad x = 4$$

	$(-\infty, \frac{5}{2})$	$(\frac{5}{2}, 4)$	$(4, \infty)$
$-4x + 10$	+	-	-
$x - 4$	-	-	+
$\frac{-4x + 10}{x - 4}$	-	+	-

The solution set: $\left\{x: x < \frac{5}{2} \cup x > 4\right\}$

9. Two companies P and Q decided to award prizes to their employees for three work ethical values, namely punctuality (x), creativity (y) and efficiency (z). Company P decided to award a total of RM3850 for the three values to 6, 2 and 3 employees respectively, while company Q decided to award RM3200 for the three values to 4, 1 and 5 employees respectively. The total amount for all the three prizes is RM1000.
- (a) Construct a system of linear equations to represent the above situation.
- (b) By forming a matrix equation, solve this equation system using the elimination method.
- (c) With the same total amount of money spent by company P and Q, is it possible for company P to award 15 employees for their creativity instead of 2 employees? Give your reason.

SOLUTION

$$\text{a) } 6x + 2y + 3z = 3850$$

$$4x + y + 5z = 3200$$

$$x + y + z = 1000$$

$$\text{b) } AX = B$$

$$\begin{pmatrix} 6 & 2 & 3 \\ 4 & 1 & 5 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3850 \\ 3200 \\ 1000 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 6 & 2 & 3 & 3850 \\ 4 & 1 & 5 & 3200 \\ 1 & 1 & 1 & 1000 \end{array} \right)$$

$$R_3 \leftrightarrow R_1 \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 4 & 1 & 5 & 3200 \\ 6 & 2 & 3 & 3850 \end{array} \right)$$

$$R_2 = 4R_1 - R_2 \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & -1 & 800 \\ 6 & 2 & 3 & 3850 \end{array} \right)$$

$$R_3 = 6R_1 - R_3 \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & -1 & 800 \\ 0 & 4 & 3 & 2150 \end{array} \right)$$

$$R_3 = 4R_2 - 3R_3 \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & -1 & 800 \\ 0 & 0 & -13 & -3250 \end{array} \right)$$

$$R_3 = \frac{1}{-13}R_3 \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & -1 & 800 \\ 0 & 0 & 1 & 250 \end{array} \right)$$

$$R_2 = R_2 + R_3 \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & 0 & 1050 \\ 0 & 0 & 1 & 250 \end{array} \right)$$

$$R_1 = R_1 - R_3 \quad \left(\begin{array}{ccc|c} 1 & 1 & 0 & 750 \\ 0 & 3 & 0 & 1050 \\ 0 & 0 & 1 & 250 \end{array} \right)$$

$$R_2 = \frac{1}{3}R_2 \quad \left(\begin{array}{ccc|c} 1 & 1 & 0 & 750 \\ 0 & 1 & 0 & 350 \\ 0 & 0 & 1 & 250 \end{array} \right)$$

$$R_1 = R_1 - R_2 \quad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 400 \\ 0 & 1 & 0 & 350 \\ 0 & 0 & 1 & 250 \end{array} \right)$$

$$\therefore x = 400, y = 350, z = 250$$

c) $6x + 15y + 3z = 3850$

$$4x + y + 5z = 3200$$

$$x + y + z = 1000$$

$$\begin{pmatrix} 6 & 15 & 3 \\ 4 & 1 & 5 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3850 \\ 3200 \\ 1000 \end{pmatrix}$$

$$\text{Let } A = \begin{pmatrix} 6 & 15 & 3 \\ 4 & 1 & 5 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{aligned} |A| &= \begin{vmatrix} 6 & 15 & 3 \\ 4 & 1 & 5 \\ 1 & 1 & 1 \end{vmatrix} \\ &= (1) \begin{vmatrix} 15 & 3 \\ 1 & 5 \end{vmatrix} - (1) \begin{vmatrix} 6 & 3 \\ 4 & 5 \end{vmatrix} + (1) \begin{vmatrix} 6 & 15 \\ 4 & 1 \end{vmatrix} \\ &= (75 - 3) - (30 - 12) + (6 - 60) \\ &= 0 \end{aligned}$$

Since $|A| = 0$, A is singular. thus the system of linear equation have no unique solution.

\therefore It is not possible for company P to award 15 employees for their creativity instead of 2 with the same total amount of money.

10. (a) Determine whether $f(x) = \frac{1}{x-4}$ and $g(x) = \frac{4x+1}{x}$ are inverse function of each other by computing their composite functions.
- (b) Given $f(x) = \ln(1 - 3x)$.
- (i) Determine the domain and range of $f(x)$. Then sketch the graph of $f(x)$.
- (ii) Find $f^{-1}(x)$, if it exists. Hence, state the domain and range of $f^{-1}(x)$.

SOLUTION

$$\text{a) } f(x) = \frac{1}{x-4} \qquad g(x) = \frac{4x+1}{x}$$

$$\begin{aligned} f[g(x)] &= f\left[\frac{4x+1}{x}\right] \\ &= \frac{1}{\left(\frac{4x+1}{x}\right) - 4} \\ &= \frac{1}{4x+1-4x} \\ &= x \end{aligned}$$

Since $f[g(x)] = x$, thus f and g are inverse function of each other.

$$\text{b) } f(x) = \ln(1 - 3x)$$

$$\text{i. } D_f: 1 - 3x > 0$$

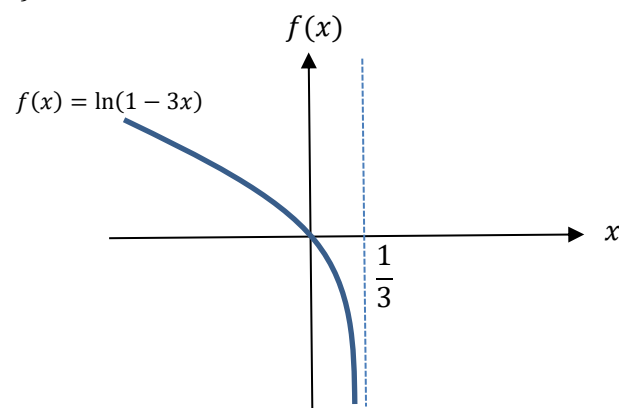
$$1 > 3x$$

$$3x < 1$$

$$x < \frac{1}{3}$$

$$D_f = \left\{x: x < \frac{1}{3}\right\}$$

$$R_f = \{y \in \mathbb{R}\}$$



$$\begin{aligned}\text{ii. } f[f^{-1}(x)] &= x \\ \ln[1 - 3f^{-1}(x)] &= x \\ 1 - 3f^{-1}(x) &= e^x \\ 3f^{-1}(x) &= 1 - e^x \\ f^{-1}(x) &= \frac{1 - e^x}{3} \\ D_{f^{-1}} &= \{x \in \mathbb{R}\} \\ R_{f^{-1}} &= \left\{y: y < \frac{1}{3}\right\}\end{aligned}$$