



**KEMENTERIAN
PENDIDIKAN
MALAYSIA**

QS 015/1

**Matriculation Programme
Examination**

Semester I

Session 2017/2018

1. Given matrix $A = \begin{pmatrix} 2 & 3 \\ -2 & 5 \end{pmatrix}$ such that $A^2 + \alpha A + \beta I = 0$, α and β are constants, where I and O are identity matrix and zero matrix of 2×2 respectively. Determine the value of α and β .
2. Solve the equation $3^{2x+1} - (16)3^x + 5 = 0$.
3. The first and three more successive terms in a geometric progression are given as follows:
 $7, \dots, 189, y, 1701, \dots$
Obtain the common ratio r . Hence, find the smallest integer n such that the n -th term exceeds 10,000.
4.
 - a) Expand $\left(1 - \frac{x}{3}\right)^{\frac{1}{2}}$ in ascending power of x up to the term in x^3 and state the interval of x for which the expansion is valid.
 - b) From part 4(a), express $\sqrt{9 - 3x}$ in the form of $a\left(1 - \frac{x}{3}\right)^{\frac{1}{2}}$, where a is an integer.
 - c) Hence, by substituting the suitable value of x , approximate $\sqrt{8.70}$ correct to two decimal places.
5. Solve the equation $3 \log_9 x = (\log_3 x)^2$.
6. Given a complex number $z = 2 + i$.
 - a. Express $\bar{z} - \frac{1}{z}$ in the form $a + bi$, where a and b are real numbers.
 - b. Obtain $\left|\bar{z} - \frac{1}{z}\right|$. Hence, determine the values of real numbers α and β if $\alpha + \beta i = \left|\bar{z} - \frac{1}{z}\right| \left(\bar{z} - \frac{1}{z}\right)^2$.
7. Find the interval of x for which the following inequalities are true.
 - a. $\frac{5}{x+3} - 1 \leq 0$.
 - b. $\left|\frac{3x-2}{2x+3}\right| > 2$.
8. Consider functions of $f(x) = (x - 2)^2 + 1, x > 2$ and $g(x) = \ln(x + 1), x > 0$.
 - a. Find $f^{-1}(x)$ and $g^{-1}(x)$, and state the domain and range for each of the inverse function.
 - b. Obtain $(g \circ f)(x)$. Hence, evaluate $(g \circ f)(2)$.
9. Given the function $g(x) = \frac{1}{2x-5}$.
 - a. Find the domain and range of $g(x)$.
 - b. Show that $g(x)$ is a one-to-one function. Hence, find $g^{-1}(x)$.
 - c. On the same axis, sketch the graph of $g(x)$ and $g^{-1}(x)$.
 - d. Show that $g \circ g^{-1}(x) = x$.
10. Given the system of linear equations as follow:

$$2x + 4y + z = 77$$

$$4x + 3y + 7z = 114$$

$$2x + y + 3z = 48$$

- a. Express the system of equations in the form of matrix equation $AX = B$ where

$x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Hence, determine matrix A and matrix B .

- b. Based on part 10(a), obtain $|A|$.

Hence, find

- i. $|P|$ if $PA = I$, where I is an identity matrix 3×3 .
- ii. $|Q|$ if $Q = (2A)^T$.
- iii. Find adjoint A .

Hence, obtain A^{-1} and find the values of x, y and z .

END OF QUESTION PAPER

1. Given matrix $A = \begin{pmatrix} 2 & 3 \\ -2 & 5 \end{pmatrix}$ such that $A^2 + \alpha A + \beta I = 0$, α and β are constants, where I and O are identity matrix and zero matrix of 2×2 respectively. Determine the value of α and β .

SOLUTION

$$A = \begin{pmatrix} 2 & 3 \\ -2 & 5 \end{pmatrix}$$

$$A^2 + \alpha A + \beta I = 0$$

$$\begin{pmatrix} 2 & 3 \\ -2 & 5 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -2 & 5 \end{pmatrix} + \alpha \begin{pmatrix} 2 & 3 \\ -2 & 5 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 - 6 & 6 + 15 \\ -4 - 10 & -6 + 25 \end{pmatrix} + \begin{pmatrix} 2\alpha & 3\alpha \\ -2\alpha & 5\alpha \end{pmatrix} + \begin{pmatrix} \beta & 0 \\ 0 & \beta \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 21 \\ -14 & 19 \end{pmatrix} + \begin{pmatrix} 2\alpha & 3\alpha \\ -2\alpha & 5\alpha \end{pmatrix} + \begin{pmatrix} \beta & 0 \\ 0 & \beta \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 + 2\alpha + \beta & 21 + 3\alpha \\ -14 - 2\alpha & 19 + 5\alpha + \beta \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$21 + 3\alpha = 0$$

$$\alpha = -7$$

$$-2 + 2\alpha + \beta = 0$$

$$-2 + 2(-7) + \beta = 0$$

$$\beta = 16$$

$$\therefore \alpha = -7, \beta = 16$$

2. Solve the equation $3^{2x+1} - (16)3^x + 5 = 0$.

SOLUTION

$$3^{2x+1} - (16)3^x + 5 = 0$$

$$3^{2x}3^1 - (16)3^x + 5 = 0$$

$$3.(3^x)^2 - (16)3^x + 5 = 0$$

$$\text{Let } y = 3^x$$

$$3y^2 - 16y + 5 = 0$$

$$(3y - 1)(y - 5) = 0$$

$$(3y - 1) = 0 \quad (y - 5) = 0$$

$$y = \frac{1}{3} \quad y = 5$$

$$3^x = \frac{1}{3} \quad 3^x = 5$$

$$3^x = 3^{-1} \quad \ln 3^x = \ln 5$$

$$x = -1 \quad x \ln 3 = \ln 5$$

$$x = 1.465$$

$$\therefore x = -1 \text{ or } x = 1.465$$

3. The first and three more successive terms in a geometric progression are given as follows:

$$7, \dots, 189, y, 1701, \dots$$

Obtain the common ratio r . Hence, find the smallest integer n such that the n -th term exceeds 10,000.

SOLUTION

$$7, \dots, 189, y, 1701, \dots$$

$$a = 7$$

$$\frac{y}{189} = \frac{1701}{y}$$

$$y^2 = 321489$$

$$y = 567$$

$$r = \frac{567}{189}$$

$$r = 3$$

$$T_n > 10000$$

$$ar^{n-1} > 10000$$

$$(7)3^{n-1} > 10000$$

$$3^{n-1} > 1428.57$$

$$\ln 3^{n-1} > \ln 1428.57$$

$$(n - 1) \ln 3 > \ln 1428.57$$

$$(n - 1) > 6.61$$

$$n > 6.61 + 1$$

$$n > 7.61$$

$$n = 8$$

4. a) Expand $\left(1 - \frac{x}{3}\right)^{\frac{1}{2}}$ in ascending power of x up to the term in x^3 and state the interval of x for which the expansion is valid.
- b) From part 4(a), express $\sqrt{9 - 3x}$ in the form of $a\left(1 - \frac{x}{3}\right)^{\frac{1}{2}}$, where a is an integer.
- c) Hence, by substituting the suitable value of x , approximate $\sqrt{8.70}$ correct to two decimal places.

SOLUTION

$$\begin{aligned}
 \text{a. } \left(1 - \frac{x}{3}\right)^{\frac{1}{2}} &= 1 + \frac{\left(\frac{1}{2}\right)}{1!} \left(-\frac{x}{3}\right)^1 + \frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)}{2!} \left(-\frac{x}{3}\right)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{3!} \left(-\frac{x}{3}\right)^3 \\
 &= 1 - \frac{x}{6} - \frac{1}{8} \left(\frac{x^2}{9}\right) - \frac{3}{48} \left(\frac{x^3}{27}\right) \\
 &= 1 - \frac{x}{6} - \frac{x^2}{72} - \frac{3}{48} \left(\frac{x^3}{27}\right) \\
 &= 1 - \frac{1}{6}x - \frac{1}{72}x^2 - \frac{1}{432}x^3
 \end{aligned}$$

The interval of x for which the expansion is valid:

$$\left|\frac{x}{3}\right| < 1$$

$$-1 < \frac{x}{3} < 1$$

$$-3 < x < 3$$

$$\begin{aligned}
 \text{b. } \sqrt{9 - 3x} &= (9 - 3x)^{\frac{1}{2}} \\
 &= 9^{\frac{1}{2}} \left(1 - \frac{3x}{9}\right)^{\frac{1}{2}} \\
 &= 3 \left(1 - \frac{x}{3}\right)^{\frac{1}{2}}
 \end{aligned}$$

$$\text{c. } \sqrt{8.70} = \sqrt{9 - 3(0.01)}$$

$$x = 0.01$$

$$\sqrt{9 - 3x} = 3 \left(1 - \frac{x}{3}\right)^{\frac{1}{2}}$$

$$\sqrt{9 - 3x} = 3 \left[1 - \frac{1}{6}x - \frac{1}{72}x^2 - \frac{1}{432}x^3\right]$$

$$\sqrt{9 - 3(0.01)} = 3 \left[1 - \frac{1}{6}(0.01) - \frac{1}{72}(0.01)^2 - \frac{1}{432}(0.01)^3\right]$$

$$\sqrt{8.7} = 2.95$$

5. Solve the equation $3 \log_9 x = (\log_3 x)^2$.

SOLUTION

$$3 \log_9 x = (\log_3 x)^2$$

$$\frac{3 \log_3 x}{\log_3 9} = (\log_3 x)^2$$

$$\frac{3 \log_3 x}{\log_3 3^2} = (\log_3 x)^2$$

$$\frac{3 \log_3 x}{2 \log_3 3} = (\log_3 x)^2$$

$$\frac{3 \log_3 x}{2} = (\log_3 x)^2$$

$$3 \log_3 x = 2(\log_3 x)^2$$

$$\text{Let } y = \log_3 x$$

$$3y = 2y^2$$

$$2y^2 - 3y = 0$$

$$y(2y - 3) = 0$$

$$y = 0$$

$$2y - 3 = 0$$

$$y = 0$$

$$y = \frac{3}{2}$$

$$\log_3 x = 0$$

$$\log_3 x = \frac{3}{2}$$

$$x = 3^0$$

$$x = 3^{\frac{3}{2}}$$

$$x = 1$$

$$x = 5.196$$

6. Given a complex number $z = 2 + i$.
- Express $\bar{z} - \frac{1}{\bar{z}}$ in the form $a + bi$, where a and b are real numbers.
 - Obtain $\left| \bar{z} - \frac{1}{\bar{z}} \right|$. Hence, determine the values of real numbers α and β if

$$\alpha + \beta i = \left| \bar{z} - \frac{1}{\bar{z}} \right| \left(\bar{z} - \frac{1}{\bar{z}} \right)^2.$$

SOLUTION

(a) $z = 2 + i$

$$\begin{aligned} \bar{z} - \frac{1}{\bar{z}} &= (2 - i) - \frac{1}{2 - i} \\ &= (2 - i) - \frac{1}{(2 - i)(2 + i)} \\ &= (2 - i) - \frac{(2 + i)}{4 + 2i - 2i - i^2} \\ &= (2 - i) - \frac{(2 + i)}{5} \\ &= 2 - i - \frac{2}{5} - \frac{1}{5}i \\ &= \frac{8}{5} - \frac{6}{5}i \end{aligned}$$

(b) $\left| \bar{z} - \frac{1}{\bar{z}} \right| = \left| \frac{8}{5} - \frac{6}{5}i \right|$

$$\begin{aligned} &= \sqrt{\left(\frac{8}{5}\right)^2 + \left(\frac{6}{5}\right)^2} \\ &= \sqrt{\frac{64}{25} + \frac{36}{25}} \\ &= \sqrt{\frac{100}{25}} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

$$\alpha + \beta i = \left| \bar{z} - \frac{1}{\bar{z}} \right| \left(\bar{z} - \frac{1}{\bar{z}} \right)^2$$

$$\alpha + \beta i = 2 \left(\frac{8}{5} - \frac{6}{5}i \right)^2$$

$$\alpha + \beta i = 2 \left[\left(\frac{8}{5} \right)^2 - 2 \left(\frac{8}{5} \right) \left(\frac{6}{5} \right) i + \left(\frac{6}{5}i \right)^2 \right]$$

$$\alpha + \beta i = 2 \left[\frac{64}{25} - \frac{96}{25}i - \frac{36}{25} \right]$$

$$\alpha + \beta i = 2 \left[\frac{28}{25} - \frac{96}{25}i \right]$$

$$\alpha + \beta i = \frac{56}{25} - \frac{192}{25}i$$

$$\alpha = \frac{56}{25} \qquad \beta = -\frac{192}{25}$$

7. Find the interval of x for which the following inequalities are true.

a. $\frac{5}{x+3} - 1 \leq 0$

b. $\left| \frac{3x-2}{2x+3} \right| > 2$

SOLUTION

a) $\frac{5}{x+3} - 1 \leq 0$

$$\frac{5 - (x + 3)}{x + 3} \leq 0$$

$$\frac{5 - x - 3}{x + 3} \leq 0$$

$$\frac{2 - x}{x + 3} \leq 0$$

$$2 - x = 0$$

$$x = 2$$

$$x + 3 = 0$$

$$x = -3$$

	$(-\infty, -3)$	$(-3, 2)$	$(2, \infty)$
$2 - x$	+	+	-
$x + 3$	-	+	+
$\frac{2 - x}{x + 3}$	-	+	-

$$\{x: x < -3 \cup x \geq 2\}$$

b) $\left| \frac{3x-2}{2x+3} \right| > 2$

$$|x| > a \Leftrightarrow x > a \text{ or } x < -a$$

$\frac{3x-2}{2x+3} > 2$ $\frac{3x-2}{2x+3} - 2 > 0$ $\frac{(3x-2) - 2(2x+3)}{2x+3} > 0$ $\frac{3x-2-4x-6}{2x+3} > 0$ $\frac{-x-8}{2x+3} > 0$	or	$\frac{3x-2}{2x+3} < -2$ $\frac{3x-2}{2x+3} + 2 < 0$ $\frac{(3x-2) + 2(2x+3)}{2x+3} < 0$ $\frac{3x-2+4x+6}{2x+3} < 0$ $\frac{7x+4}{2x+3} < 0$
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$-x - 8 = 0$ $x = -8$	$2x + 3 = 0$ $x = -\frac{3}{2}$																																
<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 15%;"></td> <td style="width: 25%;">$(-\infty, -8)$</td> <td style="width: 25%;">$(-8, -\frac{3}{2})$</td> <td style="width: 35%;">$(-\frac{3}{2}, \infty)$</td> </tr> <tr> <td>$-x - 8$</td> <td style="text-align: center;">+</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> </tr> <tr> <td>$2x + 3$</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> <td style="text-align: center;">+</td> </tr> <tr> <td>$\frac{-x - 8}{2x + 3}$</td> <td style="text-align: center;">-</td> <td style="text-align: center; background-color: #90EE90;">+</td> <td style="text-align: center;">-</td> </tr> </table>		$(-\infty, -8)$	$(-8, -\frac{3}{2})$	$(-\frac{3}{2}, \infty)$	$-x - 8$	+	-	-	$2x + 3$	-	-	+	$\frac{-x - 8}{2x + 3}$	-	+	-	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 15%;"></td> <td style="width: 25%;">$(-\infty, -\frac{3}{2})$</td> <td style="width: 25%;">$(-\frac{3}{2}, -\frac{4}{7})$</td> <td style="width: 35%;">$(-\frac{4}{7}, \infty)$</td> </tr> <tr> <td>$7x + 4$</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> <td style="text-align: center;">+</td> </tr> <tr> <td>$2x + 3$</td> <td style="text-align: center;">-</td> <td style="text-align: center;">+</td> <td style="text-align: center;">+</td> </tr> <tr> <td>$\frac{7x + 4}{2x + 3}$</td> <td style="text-align: center;">+</td> <td style="text-align: center; background-color: #90EE90;">-</td> <td style="text-align: center;">+</td> </tr> </table>		$(-\infty, -\frac{3}{2})$	$(-\frac{3}{2}, -\frac{4}{7})$	$(-\frac{4}{7}, \infty)$	$7x + 4$	-	-	+	$2x + 3$	-	+	+	$\frac{7x + 4}{2x + 3}$	+	-	+
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$(-8, -\frac{3}{2}) \cup (-\frac{3}{2}, -\frac{4}{7})$

8. Consider functions of $f(x) = (x - 2)^2 + 1, x > 2$ and $g(x) = \ln(x + 1), x > 0$.
- Find $f^{-1}(x)$ and $g^{-1}(x)$, and state the domain and range for each of the inverse function.
 - Obtain $(g \circ f)(x)$. Hence, evaluate $(g \circ f)(2)$.

SOLUTION

$$f(x) = (x - 2)^2 + 1, \quad x > 2$$
$$g(x) = \ln(x + 1), \quad x > 0.$$

(a) Let $y = f^{-1}(x)$

$$f(y) = x$$
$$(y - 2)^2 + 1 = x$$
$$(y - 2)^2 = x - 1$$
$$y - 2 = \sqrt{x - 1}$$
$$y = \sqrt{x - 1} + 2$$
$$f^{-1}(x) = \sqrt{x - 1} + 2$$

Let $y = g^{-1}(x)$

$$g(y) = x$$
$$\ln(y + 1) = x$$
$$y + 1 = e^x$$
$$y = e^x - 1$$
$$g^{-1}(x) = e^x - 1$$

$$D_{f^{-1}}: (1, \infty)$$

$$R_{f^{-1}}: (2, \infty)$$

$$D_{g^{-1}}: (0, \infty)$$

$$R_{g^{-1}}: (0, \infty)$$

(b) $(g \circ f)(x)$

$$g[f(x)] = g[(x - 2)^2 + 1]$$
$$= \ln[(x - 2)^2 + 1 + 1]$$
$$= \ln[(x - 2)^2 + 2]$$

$$(g \circ f)(2) = \ln[(2 - 2)^2 + 2]$$
$$= \ln[2]$$

9. Given the function $g(x) = \frac{1}{2x-5}$.
- Find the domain and range of $g(x)$.
 - Show that $g(x)$ is a one-to-one function. Hence, find $g^{-1}(x)$.
 - On the same axis, sketch the graph of $g(x)$ and $g^{-1}(x)$.
 - Show that $g \circ g^{-1}(x) = x$.

SOLUTION

$$g(x) = \frac{1}{2x-5}$$

(a) $D_g: 2x - 5 \neq 0$

$$x \neq \frac{5}{2}$$

$$D_g: \left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$$

$$R_g: (-\infty, 0) \cup (0, \infty)$$

(b) $g(x) = \frac{1}{2x-5}$

$$g(x_1) = \frac{1}{2x_1-5}$$

$$g(x_2) = \frac{1}{2x_2-5}$$

Let $g(x_1) = g(x_2)$

$$\frac{1}{2x_1-5} = \frac{1}{2x_2-5}$$

$$2x_1 - 5 = 2x_2 - 5$$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

Since $x_1 = x_2$ when $g(x_1) = g(x_2)$, therefore $g(x)$ is one to one function.

Let $y = g^{-1}(x)$

$$g(y) = x$$

$$\frac{1}{2y-5} = x$$

$$2y - 5 = \frac{1}{x}$$

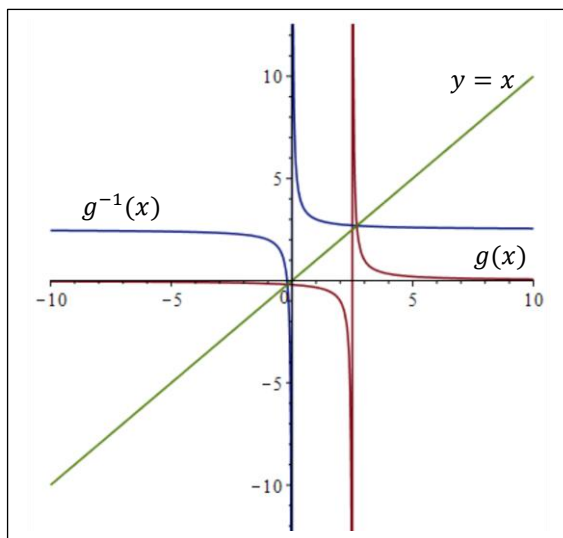
$$2y = \frac{1}{x} + 5$$

$$2y = \frac{1 + 5x}{x}$$

$$y = \frac{1 + 5x}{2x}$$

$$g^{-1}(x) = \frac{1 + 5x}{2x}$$

(c)



$$(d) g \circ g^{-1}(x) = g[g^{-1}(x)]$$

$$= g\left[\frac{1 + 5x}{2x}\right]$$

$$= \frac{1}{2\left[\frac{1 + 5x}{2x}\right] - 5}$$

$$= \frac{1}{\left[\frac{1 + 5x}{x}\right] - 5}$$

$$= \frac{1}{\left[\frac{1 + 5x - 5x}{x}\right]}$$

$$= \frac{1}{\left[\frac{1}{x}\right]}$$

$$= x$$

10. Given the system of linear equations as follow:

$$2x + 4y + z = 77$$

$$4x + 3y + 7z = 114$$

$$2x + y + 3z = 48$$

a. Express the system of equations in the form of matrix equation $AX = B$ where

$$x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \text{ Hence, determine matrix } A \text{ and matrix } B.$$

b. Based on part 10(a), obtain $|A|$.

Hence, find

i. $|P|$ if $PA = I$, where I is an identity matrix 3×3 .

ii. $|Q|$ if $Q = (2A)^T$.

iii. Find adjoint A .

Hence, obtain A^{-1} and find the values of x, y and z .

SOLUTION

$$2x + 4y + z = 77$$

$$4x + 3y + 7z = 114$$

$$2x + y + 3z = 48$$

(a)

$$\begin{pmatrix} 2 & 4 & 1 \\ 4 & 3 & 7 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 77 \\ 114 \\ 48 \end{pmatrix}$$

$$AX = B$$

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 4 & 3 & 7 \\ 2 & 1 & 3 \end{pmatrix}; \quad B = \begin{pmatrix} 77 \\ 114 \\ 48 \end{pmatrix}$$

(b)

$$\begin{aligned} |A| &= +(2) \begin{vmatrix} 3 & 7 \\ 1 & 3 \end{vmatrix} - (4) \begin{vmatrix} 4 & 7 \\ 2 & 3 \end{vmatrix} + (1) \begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} \\ &= 2(9 - 7) - 4(12 - 14) + (4 - 6) \\ &= 2(2) - 4(-2) + (-2) \\ &= 10 \end{aligned}$$

(i) $PA = I$

$P = A^{-1}$

$|P| = |A^{-1}|$

$= \frac{1}{|A|}$

$= \frac{1}{10}$

$$\text{If } AB = I \text{ then } A = B^{-1}$$

$$|A^{-1}| = \frac{1}{|A|}$$

(ii) $Q = (2A)^T$

$|Q| = |(2A)^T|$

$= 2^3 |A^T|$

$= 8|A|$

$= 8(10)$

$= 80$

$$(kA)^T = kA^T$$

$$\text{If } A \text{ is } n \times n \text{ matrix, then } |kA| = k^n |A|$$

$$|A|^T = |A|$$

(iii) $\text{adj}(A) = C^T$

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 4 & 3 & 7 \\ 2 & 1 & 3 \end{pmatrix}$$

$$\begin{aligned} \text{Cofactor, } C &= \begin{pmatrix} + \begin{vmatrix} 3 & 7 \\ 1 & 3 \end{vmatrix} & - \begin{vmatrix} 4 & 7 \\ 2 & 3 \end{vmatrix} & + \begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} \\ - \begin{vmatrix} 4 & 1 \\ 1 & 3 \end{vmatrix} & + \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} & - \begin{vmatrix} 2 & 4 \\ 2 & 1 \end{vmatrix} \\ + \begin{vmatrix} 4 & 1 \\ 3 & 7 \end{vmatrix} & - \begin{vmatrix} 2 & 1 \\ 4 & 7 \end{vmatrix} & + \begin{vmatrix} 2 & 4 \\ 4 & 3 \end{vmatrix} \end{pmatrix} \\ &= \begin{pmatrix} 2 & 2 & -2 \\ -11 & 4 & 6 \\ 25 & -10 & -10 \end{pmatrix} \end{aligned}$$

$\text{adj}(A) = C^T$

$$= \begin{pmatrix} 2 & 2 & -2 \\ -11 & 4 & 6 \\ 25 & -10 & -10 \end{pmatrix}^T$$

$$= \begin{pmatrix} 2 & -11 & 25 \\ 2 & 4 & -10 \\ -2 & 6 & -10 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$= \frac{1}{10} \begin{pmatrix} 2 & -11 & 25 \\ 2 & 4 & -10 \\ -2 & 6 & -10 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{10} & \frac{-11}{10} & \frac{25}{10} \\ \frac{2}{10} & \frac{4}{10} & \frac{-10}{10} \\ \frac{-2}{10} & \frac{6}{10} & \frac{-10}{10} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{5} & \frac{-11}{10} & \frac{5}{2} \\ \frac{1}{5} & \frac{2}{5} & -1 \\ \frac{-1}{5} & \frac{3}{5} & -1 \end{pmatrix}$$

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & \frac{-11}{10} & \frac{5}{2} \\ \frac{1}{5} & \frac{2}{5} & -1 \\ \frac{-1}{5} & \frac{3}{5} & -1 \end{pmatrix} \begin{pmatrix} 77 \\ 114 \\ 48 \end{pmatrix}$$

$$= \begin{pmatrix} 10 \\ 13 \\ 5 \end{pmatrix}$$

$$\therefore x = 10, y = 13, z = 5$$