Chapter 2: Equations, Inequalities and Absolute Values

2.3 Absolute Values

Prepared by: kwkang

Learning Outcomes

2.3 (a) State the properties of absolute values as follows:

i. $|a| \geq 0$

ii. $|-a| = |a|$

iii. $|a + b| = |b + a|$

iv. $|a - b| = |b - a|$

v. $|ab| = |a||b|$

vi. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ where $|b| \neq 0$

Learning Outcomes

2.3 (b) Solve absolute equations of these forms:

i. $|ax + b| = cx + d$

ii. $|ax + b| = |cx + d|$

iii. $|ax^2 + bx + c| = d$

Absolute Values

Properties of absolute values

i. $|a| \geq 0$

ii. $|-a| = |a|$

iii. $|a + b| = |b + a|$

iv. $|a - b| = |b - a|$

v. $|ab| = |a||b|$

vi. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ where $|b| \neq 0$

Absolute value of a, denoted by $|a|$.

Bloom: Remembering

Absolute Equation

Types of absolute equation

1. Definition

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

2. Basic definition

$$|x| = a \quad \leftrightarrow \quad x = a \quad and \quad x = -a$$

3. Inequalities

Refer Note 2.2 Inequalities

Bloom: Remembering

Example

Solve the following equations.

(a) $|x + 3| = 6$

(b) $|x + 4| = 5x + 8$

(c) $|2x - 9| = |x + 3|$

(d) $|x^2 - 6x + 4| = 4$

Bloom: Understanding

Solution

(a) $|x+3|=6$

$x+3=6$ Or $x+3=-6$ **Using basic definition**

$x=3$ $x=-9$ **Finish solving by isolating the variable.**

$|2x-9|=|x+3|$

(b) $|x+4|=5x+8$

$x+4=5x+8$ Or $x+4=-(5x+8)$ **Using basic definition**

$4x=-4$ $x+4=-5x-8$

Bloom: Understanding

Solution (Continue…)

$4x = -4$

$x = -1$

$6x = -12$

$x = -2$

Check the answers

When $x = -1$,

$|x + 4| = |(-1) + 4| = 3$

$5(-1) + 8 = 3$

$\therefore x = -1$ is a solution.

When $x = -2$,

$|x + 4| = |(-2) + 4| = 2$

$5(-2) + 8 = -2$

$\therefore x = -2$ is not a solution.

$\therefore$ *The solution is* $x = -1$.

Bloom: Understanding

Solution (Continue…)

(c) $|2x - 9| = |x + 3|$

$(|2x - 9|)^2 = (|x + 3|)^2$

Squaring both sides

$|x| = \sqrt{x^2}$

$(2x - 9)^2 = (x + 3)^2$

$4x^2 - 36x + 81 = x^2 + 6x + 9$ Quadratic expansion

$3x^2 - 42x + 72 = 0$ Rearranging to let RHS=0

$(3x - 6)(x - 12) = 0$ Factorizing to get values x

$x = 2 \quad or \quad x = 12$

Bloom: Understanding

Solution (Continue…)

(d) $|x^2 - 6x + 4| = 4$

$x^2 - 6x + 4 = 4$ Or $x^2 - 6x + 4 = -4$ **Definition**

$x^2 - 6x = 0$ $x^2 - 6x + 8 = 0$ **Solving to get values of x**

$x(x - 6) = 0$ $(x - 4)(x - 2) = 0$

$x = 0,\quad x = 6$ $x = 4,\quad x = 2$

$\therefore$ $Solution\ set\ \{0, 2, 4, 6\}$.

Bloom: Understanding

Self-check

Solve the following equations.

(a) $|4x - 3| = 9$

(b) $|2x + 1| = 4x - 3$

(c) $|5x + 2| = |8x + 11|$

(d) $|x^2 - 4x + 2| = 2$

Bloom: Applying

Answer Self-check

(a) $x = 3, \qquad x = -\frac{3}{2}$

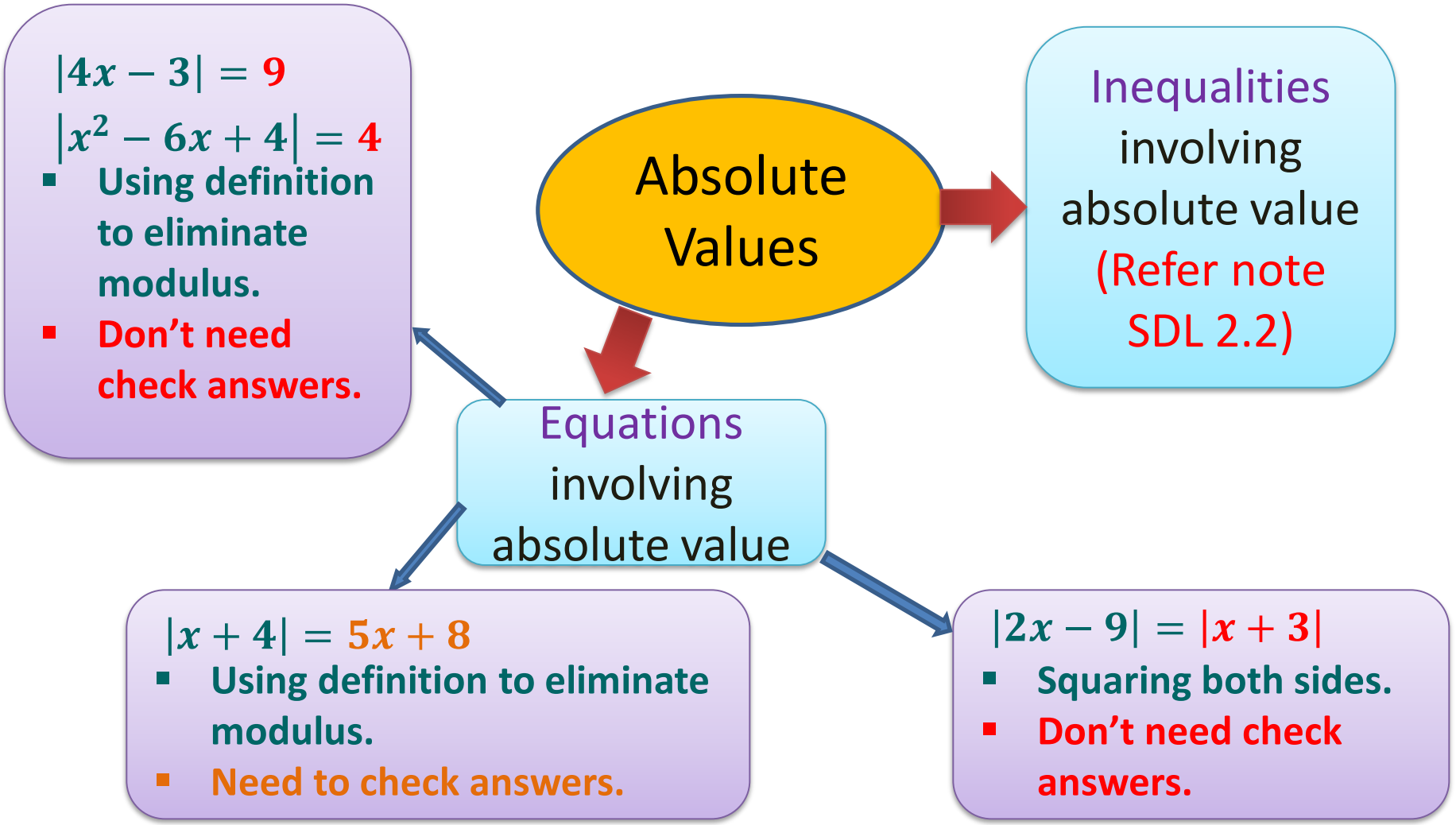
(b) $x = 2$

(c) $x = -3 \quad or \quad x = -1$

(d) $\{0, 2, 4\}.$

Bloom: Applying

Summary



Bloom: Remembering

Key Terms

- Absolute values
- Equations involving absolute value
- Squaring both sides