Chapter 2: Equations, Inequalities and Absolute Values

2.2 Inequalities

2.3 (c) Solve absolute inequalities

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Learning Outcomes

2.2 (a) Relate the properties of inequalities.

(b) Find the linear inequalities.

(c) Find the quadratic inequalities by algebraic or graphical approach.

(d) Find the rational inequalities involving linear expression.

2.3 (c) Solve absolute inequalities.

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Inequalities

Inequalities

- **Linear**
  - **Example:**
    - $x+9>4$
    - $3x-8<4$
- **Non- Linear**
  - **Example:**
    - $2x^2-x<6$
    - $\frac{6}{x}>5-x$
- **One side Modulus**
  - **Example:**
    - $|x-3|<1$
    - $|2x+5|\geq 7$
    - $\left|\frac{2x-3}{x-1}\right|\leq 2$
- **Two sides Modulus**
  - **Example:**
    - $|x+5|>|3x+3|$
    - $|5x+4|<2|x-1|$

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Bloom: Remembering

Linear Inequalities

Step of Solving Linear Inequalities

Step 1: Solve directly.

Example

Solve the following inequalities.

(a) $11 - 2x \leq 3(x+2)$

(b) $x - 20 \leq 3x - 8 < 4$

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Solution

(a) $11-2x \leq 3(x+2)$

$11-2x \leq 3x+6$ **Expanding the right hand side (RHS)**

$11-6 \leq 3x+2x$ **Collecting the x terms and the constant terms**

$5 \leq 5x$ **Finish solving by isolating the variable.**

$1 \leq x$

$x \geq 1$

$\therefore$ **Interval is** $[1, \infty)$ **.**

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Solution

(b) $x - 20 \leq 3x - 8 < 4$

Separate the inequality into two inequalities

$x - 20 \leq 3x - 8$ **and** $3x - 8 < 4$

$8 - 20 \leq 3x - x$

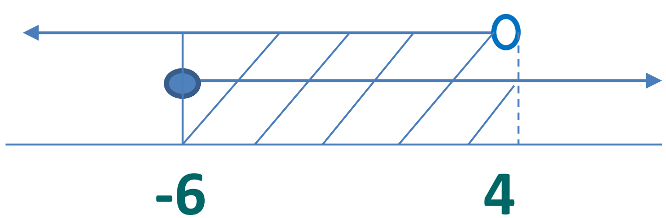
$-12 \leq 2x$

$-6 \leq x$

$3x < 4 + 8$

$3x < 12$

$x < 4$



$\therefore$ *The interval is* $[-6, 4)$.

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Non-Linear Inequalities

Steps of Solving Non-Linear Inequalities (quadratic inequalities or rational inequalities)

Step 1: Rearrange Right Hand Side (RHS) equal to **ZERO**.

Step 2: Try to **factorize till simplest** form.

Step 3: Use **TABLE OF SIGN** to determine the answer in solution set or interval form.

Example

Solve the inequality of $x^2 < 9$.

<u>Solution:</u>

$x^2 < 9$

$x^2 - 9 < 0$ Rearrange RHS=0

$(x+3)(x-3) < 0$ Factorize till simplest form

$x + 3 > 0$ and $x - 3 > 0$ Assuming both factors are positive

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Solution

Table of sign

	$x < -3$	$-3 < x < 3$	$x > 3$
$x + 3$	-	+	+
$x - 3$	-	-	+
	+	-	+

Final answer is positive or negative depends on the step before Assuming both factors are positive

$(x + 3)(x - 3) < 0$

Less than zero, so circle -

∴ **Solution set is** $\{x: -3 < x < 3\}$.

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Example

Solve the inequality of $\frac{6}{x} > 5 - x$.

<u>*Solution:*</u>

$$\frac{6}{x} > 5 - x$$

$$\frac{6}{x} - 5 + x > 0$$ **Rearrange RHS=0**

$$\frac{6 - 5x + x^2}{x} > 0$$ **Find the common denominator**

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Solution

$$\frac{x^2 - 5x + 6}{x} > 0$$

Rewrite in the general form

$$\frac{(x-2)(x-3)}{x} > 0$$

Factorize till simplest form

$x - 2 > 0$, $x - 3 > 0$, $x > 0$

Assuming all the factors are positive

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Solution

Table of sign

	$x<0$	$0<x<2$	$2<x<3$	$x>3$
x	-	+	+	+
$x-2$	-	-	+	+
$x-3$	-	-	-	+
	-	+	-	+

Final answer is positive or negative depends on the step before Assuming all factors are positive

$$\frac{(x-2)(x-3)}{x} > 0$$

Greater than zero, so circle +

$\therefore$ **Solution set is** $\{x: 0<x<2 \cup x>3\}$.

One Side Modulus Inequalities

Steps of Solving One Side Modulus Inequalities

Step 1: Rearrange the inequality in the general form.

$$|x| < a, \quad |x| > a, \quad |x| \le a, \quad |x| \ge a$$

Step 2: Eliminate the modulus by using definition.

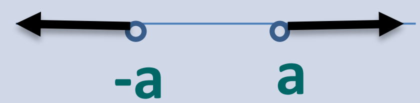
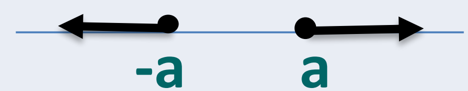
Step 3: Rearrange Right Hand Side (RHS) equal to ZERO.

Step 4: Try to factorize till simplest form.

Step 5: Use TABLE OF SIGN to determine the answer in solution set or interval form.

One Side Modulus Inequalities

Definition of modulus inequalities

Inequality	Equivalent form	Representation
$\|x\| > a$	$x > a \quad or \quad x < -a$	
$\|x\| \geq a$	$x \geq a \quad or \quad x \leq -a$	
$\|x\| < a$	$-a < x < a$	
$\|x\| \leq a$	$-a \leq x \leq a$	

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Example

Solve the following inequalities.

(a) $|x - 3| < 1$

(b) $|2x + 5| \geq 7$

(c) $\left|\frac{2x - 3}{x - 1}\right| \leq 2$

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Solution

(a) $|x - 3| < 1$

$-1 < x - 3 < 1$

Eliminating modulud by using definition.

$-1 + 3 < x < 1 + 3$

Adding 3 to both sides.

$2 < x < 4$

The solution set is $\{x: 2 < x < 4\}$

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Solution

(b) $|2x + 5| \geq 7$

$2x + 5 \geq 7$ or $2x + 5 \leq -7$

Eliminating modulus by using definition.

$2x \geq 2$ or $2x \leq -12$

$x \geq 1$ or $x \leq -6$

Solving each inequality separately.

The solution interval is $(-\infty, -6] \cup [1, \infty)$.

Solution

(c) $\left|\frac{2x-3}{x-1}\right| \leq 2$

$$-2 \leq \frac{2x-3}{x-1} \leq 2$$

Eliminating modulus by using definition.

$$-2 \leq \frac{2x-3}{x-1} \quad \text{and} \quad \frac{2x-3}{x-1} \leq 2$$

Writing double inequality as two inequalities.

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Solution (continue...)

1. Solving the first inequality:

$$-2 \leq \frac{2x-3}{x-1}$$

$$\frac{2x-3}{x-1} + 2 \geq 0$$

Rearranging RHS=0

$$\frac{2x-3+2x-2}{x-1} \geq 0$$

Combining into a single fraction.

$$\frac{4x-5}{x-1} \geq 0$$

Simplifying the numerator.

Solution (continue...)

$4x - 5 \geq 0,\qquad x - 1 > 0$

Assuming all factors greater than zero. $x - 1 \neq 0$ as it is denominator.

$x \geq \frac{5}{4},\qquad x > 1$

Table of sign

	$x < 1$	$1 < x < \frac{5}{4}$	$x \geq \frac{5}{4}$
$4x - 5$	-	+	+
$x - 1$	-	-	+
	+	-	+

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Bloom: Understanding

Solution (continue...)

Circle the positive sign or negative sign depend on the inequality before assuming all factors greater than zero.

$$\frac{4x-5}{x-1} \geq 0$$

The inequality sign greater and equal to zero, circle positive sign in the table.

$$\therefore The\ solution\ set\ is\ \left\{x: x < 1 \cup x \geq \frac{5}{4}\right\}$$

Two Sides Modulus Inequalities

Steps of Solving Two Sides Modulus Inequalities

Step 1: Squaring both sides of the inequality.
Step 2: Using the property of $(|a|)^2 = a^2$.
Step 3: Expanding both sides.
Step 4: Rearrange Right Hand Side (RHS) equal to ZERO.
Step 5: Try to factorize till simplest form.
Step 6: Use TABLE OF SIGN to determine the answer in solution set or interval form.

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Bloom: Remembering

Example

Solve the inequality $|x+5| > |3x+3|$.

Solution:

$|x+5| > |3x+3|$

$(|x+5|)^2 > (|3x+3|)^2$ **Squaring both sides of the inequality.**

$(x+5)^2 > (3x+3)^2$ **Using the property of $(|a|)^2 = a^2$.**

$x^2 + 10x + 25 > 9x^2 + 18x + 9$ **Expanding both sides.**

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Solution (continue...)

$8x^2 + 8x - 16 < 0$	Rearranging RHS=0 .
$x^2 + x - 2 < 0$	Simplifying by dividing by 8.
$(x+2)(x-1) < 0$	Factorizing till simplest form.
$x + 2 > 0$ or $x - 1 > 0$	Assuming all the factors are positive
$x > -2$ or $x > 1$	

Solution (continue…)

Table of sign

	$x < -2$	$-2 < x < 1$	$x > 1$
$x + 2$	-	+	+
$x - 1$	-	-	+
	+	-	+

$\therefore The\ solution\ set\ is\ \{x: -2 < x < 1)\}$.

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Bloom: Understanding

Self-check

(1) Solve the following inequalities. Give your answer in the interval notation.

(a) $3x + 1 \leq 5x - 9$

(b) $5x - 14 < 2x + 13 < 3(x - 2) + 25$

(2) Solve the following inequalities. Give your answer in the solution set.

(a) $x^2 - 4x < 5$ (b) $\frac{1}{2x - 1} \leq \frac{1}{2}$

Bloom: Applying

Self-check

(3) Solve each of the following inequalities.

(a) $|3x + 7| \geq 2x + 9$

(b) $\left|\dfrac{x+3}{x-2}\right| < \dfrac{1}{2}$

(4) Determine the solution set for the inequality

$|x + 5| \leq |3x - 1|$.

Bloom: Applying

Answer Self-check

(1) (a) $[\mathbf{5},\infty)$ (b) $(-\mathbf{6},\mathbf{9})$

(2) (a) $(-\mathbf{1},\mathbf{5})$ (b) $\left(-\infty,\frac{\mathbf{1}}{\mathbf{2}}\right)\cup\left[\frac{\mathbf{3}}{\mathbf{2}},\infty\right)$

(3) (a) $\left(-\infty,-\frac{\mathbf{16}}{\mathbf{5}}\right]\cup[\mathbf{2},\infty)$ (b) $\left(-\mathbf{8},-\frac{\mathbf{4}}{\mathbf{3}}\right)$

(4) $(-\infty,-\mathbf{1}]\cup[\mathbf{3},\infty)$

Bloom: Applying

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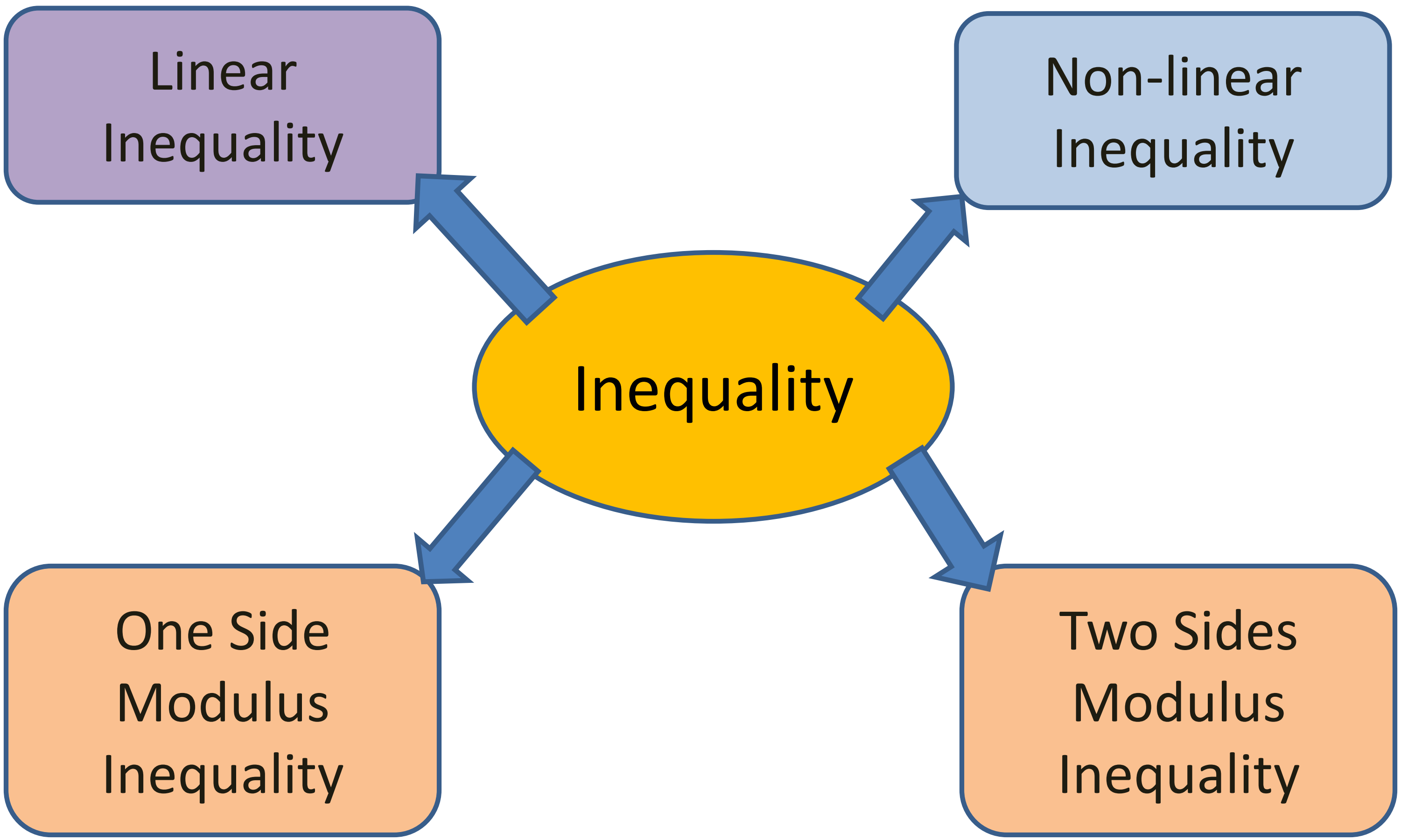
Summary



Bloom: Remembering

Key Terms

- Linear inequality
- Quadratic inequality
- Rational inequality
- One side modulus inequality
- Two sides modulus inequality
- Table of sign
- Solution set
- Interval notation

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