Chapter 1: Number System

1.2 Complex Numbers

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Learning Outcomes

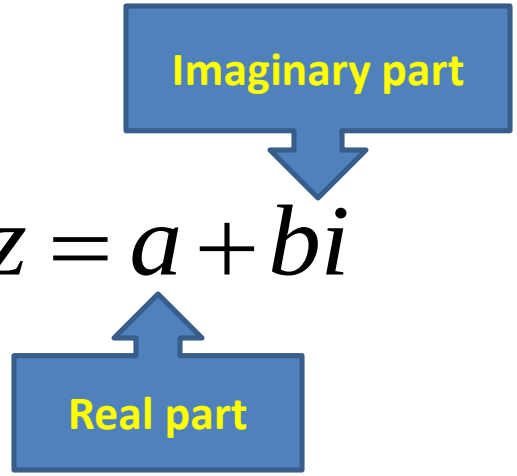
(a) Represent a complex number in Cartesian form

(b) Define the equality of two complex numbers

(c) Compute the conjugate of a Complex Number $(\bar{z})$.

(d) Determine a Complex Number in polar form $z = r(\cos\theta + i\sin\theta)$ where $r > 0$ and $-\pi < \theta < \pi$.

Complex Number



$i = \sqrt{-1}$ or $i^2 = -1$

Bloom: Remembering

Equality of two complex numbers

Real parts

$$a+bi=c+di$$

Imaginary parts

Then $a=c$ and $b=d$

Bloom: Remembering

Conjugate of a complex number

Complex number	Conjugate
$1+4i$	$1-4i$
$-5-2i$	$-5+2i$
$-3+7i$	$-3-7i$
$10i$	$-10i$

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Addition and subtraction

$z_1 = a + bi$ and $z_2 = c + di$

Addition

$$z_1 + z_2 = (a + c) + (b + d)i$$

Subtraction

$$z_1 - z_2 = (a - c) + (b - d)i$$

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Multiplication and Division

Multiplication

$$c(a+bi) = ac + bci$$

Division

$$\frac{a+bi}{c} = \frac{a}{c} + \frac{b}{c}i$$

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Example

(1) Write each of the following imaginary numbers in terms of i.

(a) $\sqrt{-4}$ (b) $\sqrt{-\frac{9}{16}}$

(2) Determine the real values of x and y if

$$x + 4 + i = 10 - (2y - 7)i$$

Bloom: Understanding

Example

(3) Find each of the following in the form $a+bi$, where a and b are real numbers.

(a) $(7+3i)+(-2+5i)$

(b) $(4-i)-(10-2i)$

(c) $(7+i)(2+5i)$

(d) $\dfrac{4}{1+2i}$

Bloom: Understanding

Solution

(1) (a) $\sqrt{-4} = \sqrt{4}i$

$= 2i$

(b) $\sqrt{-\frac{9}{16}} = \sqrt{\frac{9}{16}}i$

$= \frac{3}{4}i$

(2) $x + 4 + i = 10 - (2y - 7)i$

$(x + 4) + i = 10 + (-2y + 7)i$

$x + 4 = 10$

$\therefore x = 6$

Real part

$1 = -2y + 7$

$\therefore y = 3$

Imaginary part

Bloom: Understanding

Solution

(3) (a) $(7+3i)+(-2+5i)=(7-2)+(3i+5i)$

$=5+8i$

(b) $(4-i)-(10-2i)=(4-10)+(-i+2i)$

$=-6+i$

(c) $(7+i)(2+5i)=14+35i+2i+5i^2$

$=14+37i+5(-1)$

$=14+37i-5$

$=9+37i$

$i^2=-1$

Bloom: Understanding

Solution

(3) (d) $\dfrac{4}{1+2i} = \dfrac{4}{1+2i} \times \dfrac{1-2i}{1-2i}$

Multiply with the conjugate of the denominator

$$= \frac{4-8i}{1+4}$$

$$= \frac{4-8i}{5}$$

$$= \frac{4}{5} - \frac{8}{5}i$$

Bloom: Understanding

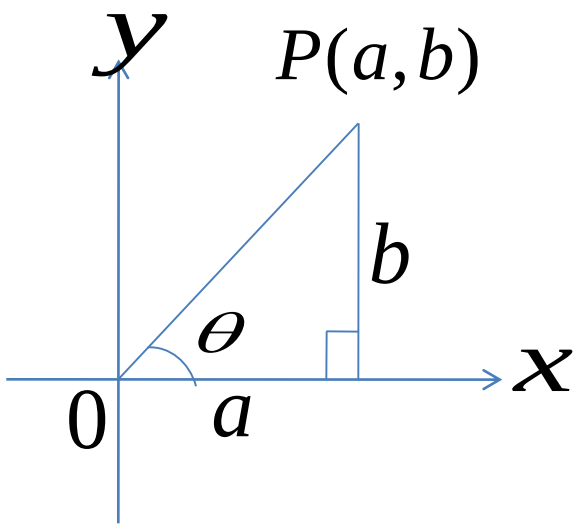
Complex number in polar form

$$z = r(\cos\theta + i\sin\theta)$$

Where,

$$r = |z| = \sqrt{a^2 + b^2}$$

Modulus of z



$$\theta = \arg(z) = \tan^{-1}\frac{b}{a}$$

Bloom: Remembering

Example

Express $\sqrt{3}+i$ in polar form.

Solution:

$$r=\sqrt{\left(\sqrt{3}\right)^2+(1)^2}=\sqrt{4}=2$$

$$\arg\left(\sqrt{3}+i\right)=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}$$

$$\therefore \theta=\frac{\pi}{6}$$

$$\therefore \sqrt{3}+i=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$$

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Self-check

(1) Simplify

(a) $-\sqrt{-81}$ (b) $\sqrt{-3}\times\sqrt{-12}$

(2) Determine the real values of x and y if

(a) $x+yi=8-x-7i$

(b) $6x+(2-y)i=-x+3yi$

(3) Express the following in the form $a+bi$

(a) $(4+i)+(3-5i)$

(b) $3(4-3i)-5(2-5i)$

Bloom: Applying

Self-check

(4) Simplify.

(a) $(-9-4i)(-3+4i)$

(b) $\left(-2+i\sqrt{2}\right)^2$

(5) Find each of the following in the Cartesian form.

(a) $\dfrac{15}{4+3i}$

(b) $\dfrac{11-2i}{-2-5i}$

Bloom: Applying

Self-check

(6) Find the modulus and argument of the complex number $z = -4 + 4\sqrt{3}i$.

Hence, express z in the polar form.

Bloom: Applying

Answer Self-check

(1) (a) $-9i$ (b) -6

(2) (a) $x = 4, y = -7$

(b) $x = 0, y = \frac{1}{2}$

(3) (a) $7-4i$ (b) $2+16i$

(4) (a) $43-24i$ (b) $2-4\sqrt{2}i$

Bloom: Applying

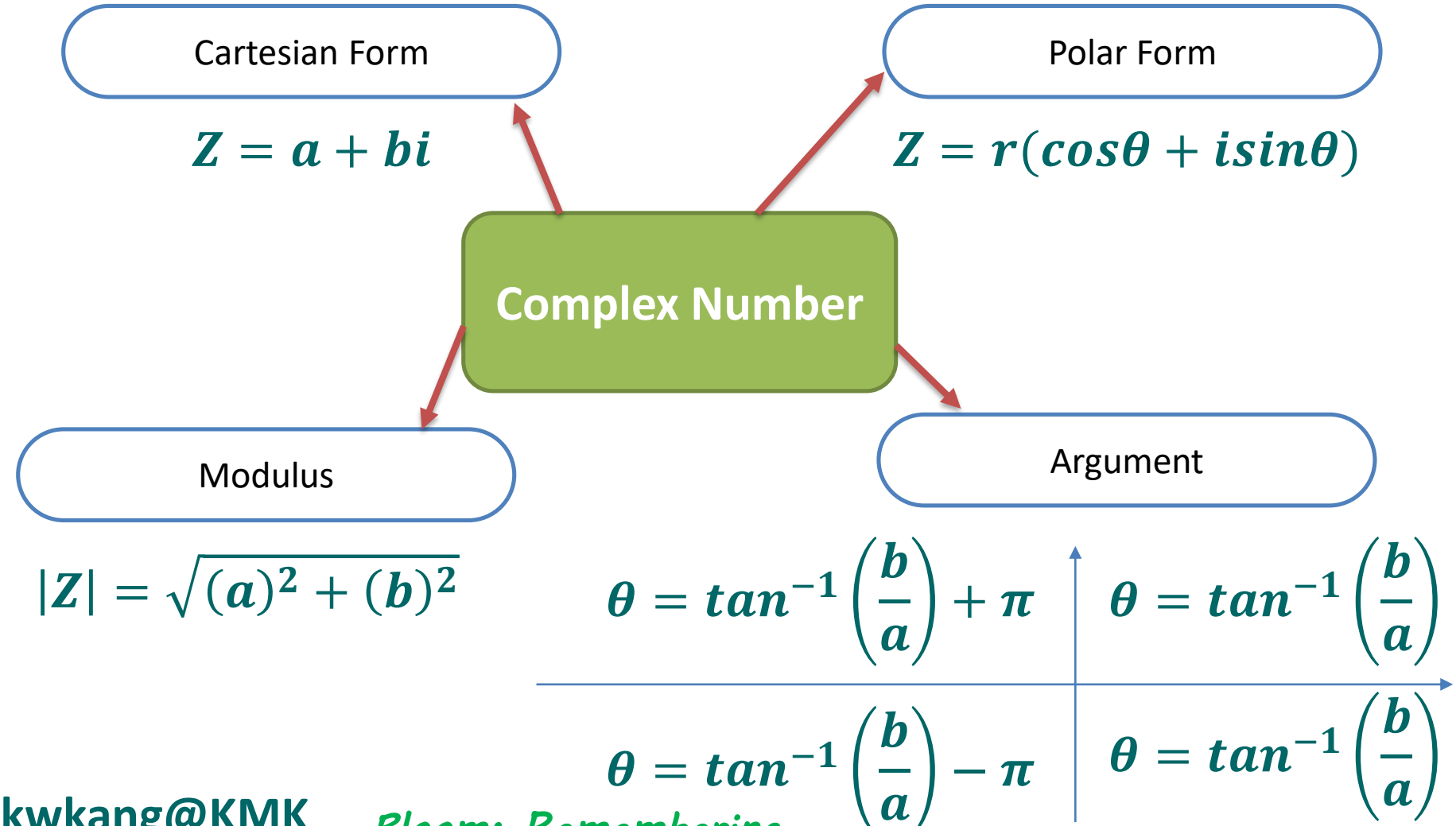
Answer Self-check

(5) (a) $\frac{12}{5} - \frac{9}{5}i$ (b) $-\frac{12}{29} + \frac{59}{29}i$

(6) $|z| = 8$, $\arg z = \frac{2\pi}{3}$, $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

Bloom: Applying

Summary



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Bloom: Remembering

Key Terms

- Real part
- Imaginary part
- Conjugate
- Modulus
- Argument
- Cartesian form
- Polar form