

SULIT
QS015/2
Mathematics
Paper 2
Semester I
Session 2017/2018
2 hours

QS015/2
Matematik
Kertas 2
Semester I
Sesi 2017/2018
2 jam



BAHAGIAN MATRIKULASI
MATRICULATION DIVISION

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

KANG KOOI WEI

Kertas soalan ini mengandungi **12** halaman bercetak.

This question paper consists of 12 printed pages.

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SULIT

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

ARAHAN KEPADA CALON:

Kertas soalan ini mengandungi 10 soalan.

Jawab semua soalan.

Semua jawapan hendaklah ditulis pada buku jawapan yang disediakan. Gunakan muka surat baharu bagi nombor soalan yang berbeza.

Markah penuh yang diperuntukkan bagi setiap soalan atau bahagian soalan ditunjukkan dalam kurungan pada penghujung soalan atau bahagian soalan.

Semua langkah kerja hendaklah ditunjukkan dengan jelas.

Kalkulator saintifik yang tidak boleh diprogramkan sahaja yang boleh digunakan.

Jawapan berangka boleh diberi dalam bentuk π , e , surd, pecahan atau sehingga tiga angka bererti, di mana-mana yang sesuai, kecuali jika dinyatakan dalam soalan.

LIST OF MATHEMATICAL FORMULAE
SENARAI RUMUS MATEMATIK

Trigonometry
Trigonometri

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

KANG KOOI WEI

LIST OF MATHEMATICAL FORMULAE
SENARAI RUMUS MATEMATIK

Differentiation
Pembezaan

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}}$$

Sphere
Sfera

$$V = \frac{4}{3} \pi r^3$$

$$S = 4 \pi r^2$$

Right circular cone
Kon membulat tegak

$$V = \frac{1}{3} \pi r^2 h$$

$$S = \pi r^2 + \pi r h$$

Right circular cylinder
Silinder membulat tegak

$$V = \pi r^2 h$$

$$S = 2\pi r^2 + 2\pi r h$$

KANG KOOI WEI

- 1 Express $\frac{3x^2 - 5}{(x-3)(x^2 + 2)}$ in partial fractions.

Ungkapkan $\frac{3x^2 - 5}{(x-3)(x^2 + 2)}$ dalam pecahan separa.

[6 marks]

[6 markah]

- 2 Solve the equation $\cos \theta + \cos 5\theta = 2\cos 3\theta$ for $0 \leq \theta \leq \pi$. Give your answers in terms of π .

Selesaikan persamaan $\cos \theta + \cos 5\theta = 2\cos 3\theta$ bagi $0 \leq \theta \leq \pi$. Beri jawapan anda dalam sebutan π .

[6 marks]

[6 markah]

- 3 Evaluate the following limits:

Nilaikan had berikut:

(a) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 2x}$.

[3 marks]

[3 markah]

(b) $\lim_{x \rightarrow \infty} \sqrt{\frac{5x+7}{6x-5}}$.

[3 marks]

[3 markah]

4 Given $y = e^{-2x} \sin 3x$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

Hence, show that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$.

Diberi $y = e^{-2x} \sin 3x$. Cari $\frac{dy}{dx}$ dan $\frac{d^2y}{dx^2}$.

Seterusnya, tunjukkan bahawa $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$.

[6 marks]

[6 markah]

5 Given the polynomial $P(x) = x^2 - 4$ and $Q(x) = \alpha x^4 + x^3 + 2x^2 + \beta x + 28$.

Diberi polinomial $P(x) = x^2 - 4$ dan $Q(x) = \alpha x^4 + x^3 + 2x^2 + \beta x + 28$.

(a) Find all zeros of $P(x)$.

Cari semua persifarah bagi $P(x)$.

[2 marks]

[2 markah]

(b) When $Q(x)$ is divided by $P(x)$, the remainder is $14x + 52$. Use the remainder theorem to find the values of α and β .

Apabila $Q(x)$ dibahagikan dengan $P(x)$, baki adalah $14x + 52$. Dengan menggunakan teorem baki, cari nilai α dan β .

[5 marks]

[5 markah]

(c) Using the values of α and β obtained from part 5(b), find the remainder when $2Q(x) + x$ is divided by $P(x)$.

Dengan menggunakan nilai α dan β yang telah diperolehi dari bahagian 5(b), cari baki apabila $2Q(x) + x$ dibahagikan dengan $P(x)$.

[5 marks]

[5 markah]

KANG KOOI WEI

- 6 Express $\cos \theta + \sqrt{2} \sin \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle.

Ungkapkan $\cos \theta + \sqrt{2} \sin \theta$ dalam bentuk $R \sin(\theta + \alpha)$, dengan $R > 0$ dan α adalah sudut tirus.

[4 marks]

[4 markah]

Hence,

Seterusnya,

- (a) solve the equation $\cos \theta + \sqrt{2} \sin \theta = \frac{\sqrt{3}}{2}$ by giving all solutions between 0° and 360° .

selesaikan persamaan $\cos \theta + \sqrt{2} \sin \theta = \frac{\sqrt{3}}{2}$ dengan memberikan semua penyelesaian antara 0° dan 360° .

[4 marks]

[4 markah]

- (b) show the greatest value of $\frac{1}{\cos \theta + \sqrt{2} \sin \theta + 5}$ is $\frac{5 + \sqrt{3}}{22}$.

tunjukkan nilai terbesar bagi $\frac{1}{\cos \theta + \sqrt{2} \sin \theta + 5}$ adalah $\frac{5 + \sqrt{3}}{22}$.

[5 marks]

[5 markah]

- 7 State the conditions for continuity of $f(x)$ at $x = a$.

Nyatakan syarat keselanjaran bagi $f(x)$ pada $x = a$.

[2 marks]

[2 markah]

- (a) By using the conditions for continuity of $f(x)$ at $x = a$, find the values of m and n such that

$$f(x) = \begin{cases} n - 2 \cos x, & x < 0 \\ 2 + mx^2, & 0 \leq x < 2 \\ m - x, & x \geq 2 \end{cases}$$

is continuous on the interval $(-\infty, \infty)$.

Dengan menggunakan syarat keselanjaran $f(x)$ pada titik $x = a$, cari nilai m dan n supaya

$$f(x) = \begin{cases} n - 2 \cos x, & x < 0 \\ 2 + mx^2, & 0 \leq x < 2 \\ m - x, & x \geq 2 \end{cases}$$

adalah selanjar pada selang $(-\infty, \infty)$.

[6 marks]

[6 markah]

- (b) If $m = -2$ and $n = 4$, determine whether $f(x)$ is differentiable at $x = 2$ or not.

Jika $m = -2$ dan $n = 4$, tentukan sama ada $f(x)$ terbeza pada $x = 2$ atau tidak.

[4 marks]

[4 markah]

- 8 A curve with equation $x^2 - 3y^2 = ae^{y-2x} + by - 6$, where a and b are constants, passes through the point $(1, 2)$.

Suatu lengkung dengan persamaan $x^2 - 3y^2 = ae^{y-2x} + by - 6$, dengan a dan b adalah pemalar, melalui titik $(1, 2)$.

- (a) Given $\frac{dy}{dx} = 1$ at $(1, 2)$, determine the values a and b .

Diberi $\frac{dy}{dx} = 1$ pada $(1, 2)$, tentukan nilai a dan b .

[8 marks]

[8 markah]

- (b) Evaluate $\frac{d^2y}{dx^2}$ at $(1, 2)$.

Nilaikan $\frac{d^2y}{dx^2}$ pada $(1, 2)$.

[4 marks]

[4 markah]

KANG KOOI WEI

- 9 The function f is defined by $f(x) = \frac{\ln(x-1)}{x-1}$ for $x > 1$.

Fungsi f ditakrifkan sebagai $f(x) = \frac{\ln(x-1)}{x-1}$ untuk $x > 1$.

- (a) By considering the first and second derivatives of $f(x)$, show that there is only one maximum point on the graph $y = f(x)$.

Dengan mempertimbangkan terbitan pertama dan terbitan kedua bagi $f(x)$, tunjukkan bahawa hanya ada satu titik maksimum pada graf $y = f(x)$.

[8 marks]

[8 markah]

- (b) Use the result obtained in part 9(a) to state the exact coordinates of the maximum point.

Gunakan keputusan yang diperolehi dalam bahagian 9(a) untuk menyatakan koordinat tepat bagi titik maksimum tersebut.

[2 marks]

[2 markah]

- (c) Find the x -coordinate of the function f when $\frac{d^2y}{dx^2} = 0$.

Cari koordinat- x bagi fungsi f apabila $\frac{d^2y}{dx^2} = 0$.

[2 marks]

[2 markah]

- 10 A curve is defined by the parametric equations $x = 3t - \frac{1}{t}$ and $y = t + \frac{3}{t}$, where $t \neq 0$.

Suatu lengkung ditakrifkan oleh persamaan berparameter $x = 3t - \frac{1}{t}$ dan $y = t + \frac{3}{t}$, dengan $t \neq 0$.

- (a) Show that $\frac{dy}{dx} = \frac{t^2 - 3}{3t^2 + 1}$. Hence, find $\frac{d^2y}{dx^2}$.

Tunjukkan $\frac{dy}{dx} = \frac{t^2 - 3}{3t^2 + 1}$. Seterusnya, cari $\frac{d^2y}{dx^2}$.

[8 marks]

[8 markah]

- (b) Show that $\frac{dy}{dx}$ can be expressed as $\frac{dy}{dx} = \frac{1}{3} - \frac{10}{3(3t^2 + 1)}$.

Hence, deduce that $-3 < \frac{dy}{dx} < \frac{1}{3}$.

Tunjukkan $\frac{dy}{dx}$ boleh diungkap sebagai $\frac{dy}{dx} = \frac{1}{3} - \frac{10}{3(3t^2 + 1)}$.

Seterusnya, deduksikan bahawa $-3 < \frac{dy}{dx} < \frac{1}{3}$.

[7 marks]

[7 markah]

END OF QUESTION PAPER

KERTAS SOALAN TAMAT

KANG KOOI WEI