

QS 015/1
Matriculation Programme
Examination
Semester I
Session 2016/2017

1. Solve for p and q where $p \neq q$, such that $\frac{(p+qi)}{3i} = (3 + \sqrt{-16} - i^3)$.

SOLUTION

$$\frac{(p + qi)}{3i} = (3 + \sqrt{-16} - i^3)$$

$$p + qi = (3 + \sqrt{-16} - i^3)(3i)$$

$$p + qi = (3 + \sqrt{16}\sqrt{-1} - i^3)(3i)$$

$$p + qi = (3 + 4i - i^3)(3i)$$

$$p + qi = 9i + 12i^2 - 3i^4$$

$$p + qi = 9i + 12(-1) - 3(1)$$

$$p + qi = 9i - 15$$

$$p + qi = -15 + 9i$$

$$\mathbf{p = -15, \quad q = 9}$$

2. Determine the values of x which satisfy the equation $3^{2x-1} = 4(3^x) - 9$.

SOLUTION

$$3^{2x-1} = 4(3^x) - 9$$

$$3^{2x}3^{-1} = 4(3^x) - 9$$

$$(3^x)^2 \left(\frac{1}{3}\right) = 4(3^x) - 9$$

$$\left(\frac{1}{3}\right)(3^x)^2 = 4(3^x) - 9$$

$$(3^x)^2 = 12(3^x) - 27$$

Let $y = 3^x$

$$(y)^2 = 12(y) - 27$$

$$y^2 - 12y + 27 = 0$$

$$(y - 3)(y - 9) = 0$$

$$y = 3 \qquad y = 9$$

$$3^x = 3 \qquad 3^x = 9$$

$$x = 1 \qquad x = 2$$

3. The seventh term of a geometric series is 16, the fifth term is 8 and the sum of the first ten terms is positive. Find the first term and the common ratio. Hence, show that $S_{12} = 126(\sqrt{2} + 1)$.

SOLUTION**Geometric series**

Geometric series $\rightarrow T_n = ar^{n-1}$

$$T_7 = ar^6 = 16 \quad \dots\dots (1)$$

$$T_5 = ar^4 = 8 \quad \dots\dots (2)$$

$$(1) \div (2)$$

$$\frac{ar^6}{ar^4} = \frac{16}{8}$$

$$r^2 = 2$$

$$r = \pm\sqrt{2}$$

When $r = \pm\sqrt{2}$

$$a(\pm\sqrt{2})^4 = 8$$

$$4a = 8$$

$$a = 2$$

Geometric series $\rightarrow S_n = \frac{a(r^n - 1)}{r - 1}$

Given that $S_n > 0$

Given that $S_n > 0$

$$S_{10} = \frac{a(r^{10} - 1)}{r - 1}$$

When $a = 2$; $r = \sqrt{2}$

$$\begin{aligned} S_{10} &= \frac{2[(\sqrt{2})^{10} - 1]}{\sqrt{2} - 1} \\ &= \frac{2[32 - 1]}{\sqrt{2} - 1} \\ &= \frac{62}{\sqrt{2} - 1} \\ &= 149.68 \end{aligned}$$

When $a = 2$; $r = -\sqrt{2}$

$$\begin{aligned} S_{10} &= \frac{2[(-\sqrt{2})^{10} - 1]}{-\sqrt{2} - 1} \\ &= \frac{2[32 - 1]}{-\sqrt{2} - 1} \\ &= \frac{62}{-\sqrt{2} - 1} \\ &= -25.68 \end{aligned}$$

Given that $S_n > 0$, $\rightarrow a = 2$ $r = \sqrt{2}$

$$\begin{aligned} S_{12} &= \frac{2[(\sqrt{2})^{12} - 1]}{\sqrt{2} - 1} \\ &= \frac{2[64 - 1]}{\sqrt{2} - 1} \\ &= \frac{126}{\sqrt{2} - 1} \\ &= \frac{126}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} \\ &= \frac{126(\sqrt{2} + 1)}{2 + \sqrt{2} - \sqrt{2} - 1} \\ &= \frac{126(\sqrt{2} + 1)}{1} \\ &= 126(\sqrt{2} + 1) \end{aligned}$$

4. If A and B are 2×2 matrices where $B \neq I_2$, such that $(A + B)^2 = A^2 + 2AB + B^2$, deduce that $B = A^{-1}$ [Assume $AB = BA = I$]. If $A = \begin{bmatrix} 1 & 2 \\ 9 & -1 \end{bmatrix}$, find B .

SOLUTION

$$(A + B)^2 = A^2 + 2AB + B^2$$

$$(A + B)(A + B) = A^2 + 2AB + B^2$$

$$A^2 + AB + BA + B^2 = A^2 + 2AB + B^2$$

$$AB + BA = 2AB$$

$$BA = 2AB - AB$$

$$BA = AB$$

$$\therefore B = A^{-1}$$

If $AB = BA = I$ then $B^{-1} = A$

$$A = \begin{bmatrix} 1 & 2 \\ 9 & -1 \end{bmatrix}$$

$$B = A^{-1} = \frac{1}{(1)(-1) - (2)(9)} \begin{bmatrix} -1 & -2 \\ -9 & 1 \end{bmatrix}$$

$$= \frac{1}{-19} \begin{bmatrix} -1 & -2 \\ -9 & 1 \end{bmatrix}$$

$$= \frac{1}{-19} \begin{bmatrix} -1 & -2 \\ -9 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{19} & \frac{2}{19} \\ \frac{9}{19} & \frac{1}{19} \end{bmatrix}$$

5. (a) Obtain the expansion for $\left(1 - \frac{x}{4}\right)^{\frac{1}{4}}$ in ascending powers of x up to the term x^3 .

State the interval for x such that the expansion $\left(1 - \frac{x}{4}\right)^{\frac{1}{4}}$ is valid. Hence, obtain the simplest form of the expansion $(16 - 4x)^{\frac{1}{4}}$.

- (b) Write $\sqrt[4]{12}$ in the form of $K \left[\left(1 - \frac{x}{4}\right)^{\frac{1}{4}} \right]$. Hence, approximate $\sqrt[4]{12}$ correct to three decimal places.

SOLUTION

$$\begin{aligned}
 \text{a) } \left(1 - \frac{x}{4}\right)^{\frac{1}{4}} &= 1 + \frac{\left(\frac{1}{4}\right)}{1!} \left(-\frac{x}{4}\right)^1 + \frac{\left(\frac{1}{4}\right)\left(\frac{1}{4}-1\right)}{2!} \left(-\frac{x}{4}\right)^2 + \frac{\left(\frac{1}{4}\right)\left(\frac{1}{4}-1\right)\left(\frac{1}{4}-2\right)}{3!} \left(-\frac{x}{4}\right)^3 + \dots \\
 &= 1 + \left(\frac{1}{4}\right) \left(-\frac{x}{4}\right) + \frac{\left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)}{2 \times 1} \left(\frac{x^2}{16}\right) + \frac{\left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)}{3 \times 2 \times 1} \left(-\frac{x^3}{64}\right) + \dots \\
 &= 1 - \frac{x}{16} - \frac{3}{32} \left(\frac{x^2}{16}\right) - \frac{21}{384} \left(\frac{x^3}{64}\right) + \dots \\
 &= 1 - \frac{1}{16}x - \frac{3}{512}x^2 - \frac{7}{8192}x^3 + \dots
 \end{aligned}$$

The interval for x such that the expansion $\left(1 - \frac{x}{4}\right)^{\frac{1}{4}}$ is valid

$$\left|\frac{x}{4}\right| < 1$$

$$-1 < \frac{x}{4} < 1$$

$$-4 < x < 4$$

The simplest form of the expansion $(16 - 4x)^{\frac{1}{4}}$

$$\begin{aligned}
 (16 - 4x)^{\frac{1}{4}} &= \left[16 \left(1 - \frac{4x}{16}\right)\right]^{\frac{1}{4}} \\
 &= 16^{\frac{1}{4}} \left(1 - \frac{x}{4}\right)^{\frac{1}{4}} \\
 &= 2 \left(1 - \frac{x}{4}\right)^{\frac{1}{4}} \\
 &= 2 \left[1 - \frac{1}{16}x - \frac{3}{512}x^2 - \frac{7}{8192}x^3 + \dots\right] \\
 &= 2 - \frac{1}{8}x - \frac{3}{256}x^2 - \frac{7}{4096}x^3 + \dots
 \end{aligned}$$

b) $\sqrt[4]{12}$ in the form of $K \left[\left(1 - \frac{x}{4}\right)^{\frac{1}{4}} \right]$

$$\begin{aligned}
 \sqrt[4]{12} &= 12^{\frac{1}{4}} \\
 &= (16 - 4)^{\frac{1}{4}} \\
 &= \left[16 \left(1 - \frac{4}{16}\right)\right]^{\frac{1}{4}} \\
 &= 16^{\frac{1}{4}} \left(1 - \frac{1}{4}\right)^{\frac{1}{4}} \\
 &= 2 \left(1 - \frac{1}{4}\right)^{\frac{1}{4}}
 \end{aligned}$$

Compare

$$2 \left(1 - \frac{x}{4}\right)^{\frac{1}{4}} = 2 - \frac{1}{8}x - \frac{3}{256}x^2 - \frac{7}{4096}x^3$$

$$= 2 - \frac{1}{8}(1) - \frac{3}{256}(1)^2 - \frac{7}{4096}(1)^3 + \dots$$

$$\approx 1.862$$

6. Given $f(x) = \frac{x^2+1}{5}$, $x \geq 0$.
- (a) Determine $f^{-1}(x)$. Hence, if $f(g(x)) = \frac{1}{5}(e^{2(3x-1)} + 1)$, show that $g(x) = e^{3x-1}$
- (b) Evaluate $g(f(2))$ correct to three decimal places.
- (c) Assume that the domain for $g(x)$ is $x \geq 0$, determine $g^{-1}(x)$ and state its domain and range.

SOLUTION

a) $f(x) = \frac{x^2+1}{5}$, $x \geq 0$

Method I	Method II
$f(x) = \frac{x^2 + 1}{5}$	$y = \frac{x^2 + 1}{5}$
$f[f^{-1}(x)] = x$	$5y = x^2 + 1$
$\frac{[f^{-1}(x)]^2 + 1}{5} = x$	$x^2 = 5y - 1$
$[f^{-1}(x)]^2 + 1 = 5x$	$x = \sqrt{5y - 1}$
$[f^{-1}(x)]^2 = 5x - 1$	$f^{-1}(x) = \sqrt{5x - 1}$
$f^{-1}(x) = \sqrt{5x - 1}$	

$$f(g(x)) = \frac{1}{5}(e^{2(3x-1)} + 1)$$

$$\frac{[g(x)]^2 + 1}{5} = \frac{1}{5}(e^{2(3x-1)} + 1)$$

$$[g(x)]^2 + 1 = e^{2(3x-1)} + 1$$

$$[g(x)]^2 = e^{2(3x-1)}$$

$$g(x) = [e^{2(3x-1)}]^{\frac{1}{2}}$$

$$g(x) = e^{3x-1}$$

b) Evaluate $g(f(2))$

$$f(2) = \frac{2^2 + 1}{5}$$

$$= 1$$

$$g(f(2)) = g(1)$$

$$= e^{3(1)-1}$$

$$= e^2$$

c) $g(x) = e^{3x-1}$

Method I	Method II
$g(x) = e^{3x-1}$	$g(x) = e^{3x-1}$
$g[g^{-1}(x)] = x$	$y = e^{3x-1}$
$e^{3[g^{-1}(x)]-1} = x$	$\ln y = \ln e^{3x-1}$
$\ln e^{3[g^{-1}(x)]-1} = \ln x$	$\ln y = 3x - 1$
$3[g^{-1}(x)] - 1 = \ln x$	$3x = \ln y + 1$
$3[g^{-1}(x)] = \ln x + 1$	$x = \frac{\ln y + 1}{3}$
$g^{-1}(x) = \frac{\ln x + 1}{3}$	$g^{-1}(x) = \frac{\ln x + 1}{3}$

$$g(x) = e^{3x-1}; x \geq 0$$

$$D_{g(x)}: x \geq 0$$

$$R_{g(x)}: y \geq e^{3(0)-1}$$

$$y \geq e^{-1}$$

$$y \geq \frac{1}{e}$$

$$D_{g^{-1}(x)} = R_{g(x)}$$

$$R_{g^{-1}(x)} = D_{g(x)}$$

$$D_{g^{-1}(x)}: x \geq \frac{1}{e}$$

$$R_{g^{-1}(x)}: y \geq 0$$

7. (a) If $\sqrt{7 - 3\sqrt{5}} = \sqrt{x} - \sqrt{y}$, determine the values of x and y .
- (b) Solve the equation $\log_2 x - \log_4(3x + 4) = 0$.

SOLUTION

$$\text{a) } \sqrt{7 - 3\sqrt{5}} = \sqrt{x} - \sqrt{y}$$

$$\left[\sqrt{7 - 3\sqrt{5}} \right]^2 = [\sqrt{x} - \sqrt{y}]^2$$

$$7 - 3\sqrt{5} = \sqrt{x}^2 + \sqrt{y}^2 - 2\sqrt{x}\sqrt{y}$$

$$7 - 3\sqrt{5} = x + y - 2\sqrt{xy}$$

$$x + y = 7$$

$$y = 7 - x \quad \dots\dots\dots (1)$$

$$2\sqrt{xy} = 3\sqrt{5}$$

$$\sqrt{4xy} = \sqrt{9 \times 5}$$

$$4xy = 45 \quad \dots\dots\dots (2)$$

Substitute (1) into (2)

$$4x(7 - x) = 45$$

$$28x - 4x^2 = 45$$

$$4x^2 - 28x + 45 = 0$$

$$(2x - 9)(2x - 5) = 0$$

$$x = \frac{9}{2}$$

$$x = \frac{5}{2}$$

$$y = 7 - \frac{9}{2} \qquad y = 7 - \frac{5}{2}$$

$$y = \frac{5}{2} \qquad y = \frac{9}{2}$$

$$\therefore x = \frac{9}{2}, y = \frac{5}{2} \text{ or } x = \frac{5}{2}, y = \frac{9}{2}$$

b) $\log_2 x - \log_4(3x + 4) = 0$

$$\log_2 x - \frac{\log_2(3x + 4)}{\log_2 4} = 0$$

$$\log_2 x - \frac{\log_2(3x + 4)}{\log_2 2^2} = 0$$

$$\log_2 x - \frac{\log_2(3x + 4)}{2\log_2 2} = 0$$

$$\log_2 x - \frac{\log_2(3x + 4)}{2(1)} = 0$$

$$\log_2 x - \frac{1}{2}\log_2(3x + 4) = 0$$

$$\log_2 x = \frac{1}{2}\log_2(3x + 4)$$

$$\log_2 x = \log_2(3x + 4)^{\frac{1}{2}}$$

$$x = (3x + 4)^{\frac{1}{2}}$$

$$x^2 = 3x + 4$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4 \text{ or } x = -1$$

Since $x > 0$, $\therefore x = 4$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

$$\log b^n = n \log b$$

$$\log_a a = 1$$

8. (a) Solve the following equation $\left| \frac{3}{x-4} \right| = 7, x \neq 4$.
- (b) Find the solution set for the inequality, $\frac{-4-x}{x-3} \geq x + 4, x \neq 3$.

SOLUTION

$$(a) \left| \frac{3}{x-4} \right| = 7$$

$$\frac{3}{x-4} = 7 \quad \text{or} \quad \frac{3}{x-4} = -7$$

$$3 = 7(x - 4) \quad \quad \quad 3 = -7(x - 4)$$

$$3 = 7x - 28 \quad \quad \quad 3 = -7x + 28$$

$$7x = 3 + 28 \quad \quad \quad 7x = 28 - 3$$

$$7x = 31 \quad \quad \quad 7x = 25$$

$$x = \frac{31}{7} \quad \quad \quad x = \frac{25}{7}$$

$$(b) \frac{-4-x}{x-3} \geq x + 4, \quad x \neq 3$$

$$\frac{-4-x}{x-3} \geq x + 4$$

$$\frac{-4-x}{x-3} - x - 4 \geq 0$$

$$\frac{-4-x-x(x-3)-4(x-3)}{x-3} \geq 0$$

$$\frac{-4-x-x^2+3x-4x+12}{x-3} \geq 0$$

$$\frac{-x^2-2x+8}{x-3} \geq 0$$

$$\frac{-(x^2 + 2x - 8)}{x - 3} \geq 0$$

$$\frac{(x^2 + 2x - 8)}{x - 3} \leq 0$$

$$\frac{(x + 4)(x - 2)}{x - 3} \leq 0$$

$$(x + 4) = 0$$

$$(x - 2) = 0$$

$$(x - 3) = 0$$

$$x = -4$$

$$x = 2$$

$$x = 3$$

	$(-\infty, -4)$	$(-4, 2)$	$(2, 3)$	$(3, \infty)$
$(x + 4)$	-	+	+	+
$(x - 2)$	-	-	+	+
$(x - 3)$	-	-	-	+
$\frac{(x + 4)(x - 2)}{x - 3}$	⊖	+	⊖	+

\therefore Solution set; $\{x: -\infty < x \leq -4 \cup 2 \leq x < 3\}$

9. Given $f(x) = e^x$ and $g(x) = |x - 3|$.

(a) Show that $(f \circ g)(x) = \begin{cases} e^{x-3}, & x \geq 3 \\ e^{-(x-3)}, & x < 3 \end{cases}$

(b) Sketch the graph of $y = (f \circ g)(x)$. Hence, state the interval in which $(f \circ g)^{-1}(x)$ exists.

(c) Determine $(f \circ g)^{-1}(x)$, for $x \geq 3$.

(d) Find the function $h(x)$ for $x > \frac{1}{3}$, given that $(h \circ f)(x) = \frac{2e^x}{1-3e^x}$. Hence, show that $h(x)$ is a one to one function.

SOLUTION

$$f(x) = e^x \text{ and } g(x) = |x - 3|$$

a) $(f \circ g)(x) = f(|x - 3|)$

$$= e^{|x-3|}$$

$$|x - 3| = \begin{cases} x - 3, & x - 3 \geq 0 \\ -(x - 3), & x - 3 < 0 \end{cases}$$

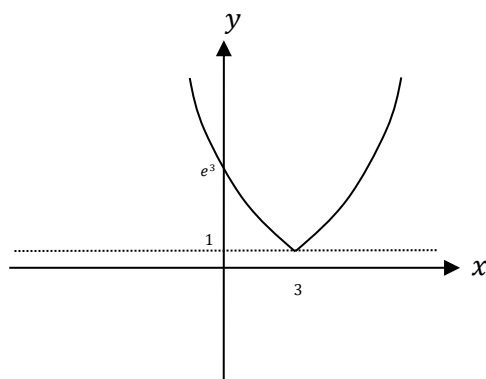
$$= \begin{cases} x - 3, & x \geq 3 \\ -(x - 3), & x < 3 \end{cases}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$(f \circ g)(x) = e^{|x-3|}$$

$$= \begin{cases} e^{x-3}, & x \geq 3 \\ e^{-(x-3)}, & x < 3 \end{cases}$$

b)



$(f \circ g)^{-1}(x)$ exists for $x \geq 3$ or $x < 3$

c) $(f \circ g)^{-1}(x)$ for $x \geq 3$

$$(f \circ g)(x) = e^{x-3}$$

$$y = e^{x-3}$$

$$\ln y = \ln e^{x-3}$$

$$\ln y = x - 3$$

$$x = \ln y + 3$$

$$\therefore (f \circ g)^{-1}(x) = \ln x + 3$$

d) $f(x) = e^x$,

$$(h \circ f)(x) = \frac{2e^x}{1 - 3e^x}$$

$$h[f(x)] = \frac{2e^x}{1 - 3e^x}$$

$$h[e^x] = \frac{2e^x}{1 - 3e^x}$$

Let $y = e^x$

$$h[y] = \frac{2y}{1 - 3y}$$

$$h[x] = \frac{2x}{1 - 3x}$$

Show that $h(x)$ is a one to one function

$$h(x) = \frac{2x}{1 - 3x}$$

Let $h(x_1) = h(x_2)$

$$\frac{2x_1}{1-3x_1} = \frac{2x_2}{1-3x_2}$$

$$2x_1(1-3x_2) = 2x_2(1-3x_1)$$

$$x_1(1-3x_2) = x_2(1-3x_1)$$

$$x_1 - 3x_1x_2 = x_2 - 3x_1x_2$$

$$x_1 = x_2$$

Since $x_1 = x_2$, therefore $h(x)$ is one to one function

10. (a) Given $P = \begin{bmatrix} 2 & 0 & -4 \\ -1 & 6 & 2 \end{bmatrix}$, $Q = \begin{bmatrix} -1 & 3 \\ 0 & 2 \\ -6 & 5 \end{bmatrix}$ and $R = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 3 & 4 & 1 \end{bmatrix}$.

Find R^{-1} by using the elementary row operations method.

Hence, if $RX = 3Q + P^T$, determine the matrix X .

(b) Ahmad bought an examination pad, 2 pens and a tube of liquid paper for RM18. Ali spent RM24 for 3 examination pads, 2 pens and 2 tubes of liquid paper. In the meantime Abu spent RM36 at the same store for 3 examination pads, 4 pens and a tube of liquid paper. Let x , y and z represent the price per unit for examination pad, pen and tube of liquid paper respectively.

- Obtain the system of linear equations from the above information.
- Write the system in the form of matrix equation $AX = B$.
- State the price of each unit of examination pad, pen and tube of liquid paper.
- Aminah bought 4 examination pads, 5 pens and 1 tube of liquid paper. What is the total price paid?

SOLUTION

a) $P = \begin{bmatrix} 2 & 0 & -4 \\ -1 & 6 & 2 \end{bmatrix}$, $Q = \begin{bmatrix} -1 & 3 \\ 0 & 2 \\ -6 & 5 \end{bmatrix}$ and $R = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 2 & 2 & 0 & 1 & 0 \\ 3 & 4 & 1 & 0 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{R_2^* = 3R_1 - R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 4 & 1 & 3 & -1 & 0 \\ 3 & 4 & 1 & 0 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{R_3^* = 3R_1 - R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 4 & 1 & 3 & -1 & 0 \\ 0 & 2 & 2 & 3 & 0 & -1 \end{array} \right)$$

$$\xrightarrow{R_2^* = \frac{1}{4}R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & 2 & -2 & 3 & 0 & -1 \end{array} \right)$$

$$\xrightarrow{R_3^* = 2R_2 - R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & -\frac{3}{2} & -\frac{3}{2} & -\frac{1}{2} & 1 \end{array} \right)$$

$$\xrightarrow{R_3^* = -\frac{2}{3}R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & 1 & 1 & \frac{1}{3} & -\frac{2}{3} \end{array} \right)$$

$$\xrightarrow{R_2^* = R_2 - \frac{1}{4}R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 1 & 1 & \frac{1}{3} & -\frac{2}{3} \end{array} \right)$$

$$\xrightarrow{R_1^* = R_1 - R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 1 & 1 & \frac{1}{3} & -\frac{2}{3} \end{array} \right)$$

$$\xrightarrow{R_1^* = R_1 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 1 & 1 & \frac{1}{3} & -\frac{2}{3} \end{array} \right)$$

$$R^{-1} = \begin{bmatrix} -1 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \\ 1 & \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$$

$$RX = 3Q + P^T$$

$$R^{-1}RX = R^{-1}(3Q + P^T)$$

$$X = R^{-1}(3Q + P^T)$$

$$\begin{aligned}
&= \begin{bmatrix} -1 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \\ 1 & \frac{1}{3} & -\frac{2}{3} \end{bmatrix} \left[3 \begin{bmatrix} -1 & 3 \\ 0 & 2 \\ -6 & 5 \end{bmatrix} + \left(\begin{bmatrix} 2 & 0 & -4 \\ -1 & 6 & 2 \end{bmatrix} \right)^T \right] \\
&= \begin{bmatrix} -1 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \\ 1 & \frac{1}{3} & -\frac{2}{3} \end{bmatrix} \left[\begin{bmatrix} -3 & 9 \\ 0 & 6 \\ -18 & 15 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 0 & 6 \\ -4 & 2 \end{bmatrix} \right] \\
&= \begin{bmatrix} -1 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \\ 1 & \frac{1}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} -1 & 8 \\ 0 & 12 \\ -22 & 17 \end{bmatrix} \\
&= \begin{bmatrix} (-1)(-1) + \left(\frac{1}{3}\right)(0) + \left(\frac{1}{3}\right)(-22) & (-1)(8) + \left(\frac{1}{3}\right)(12) + \left(\frac{1}{3}\right)(17) \\ \left(\frac{1}{2}\right)(-1) + \left(-\frac{1}{3}\right)(0) + \left(\frac{1}{6}\right)(-22) & \left(\frac{1}{2}\right)(8) + \left(-\frac{1}{3}\right)(12) + \left(\frac{1}{6}\right)(17) \\ (1)(-1) + \left(\frac{1}{3}\right)(0) + \left(-\frac{2}{3}\right)(-22) & (1)(8) + \left(\frac{1}{3}\right)(12) + \left(-\frac{2}{3}\right)(17) \end{bmatrix} \\
&= \begin{bmatrix} 1 + 0 - \frac{22}{3} & -8 + \frac{12}{3} + \frac{17}{3} \\ -\frac{1}{2} + 0 - \frac{22}{6} & 4 - 4 + \frac{17}{6} \\ -1 + 0 + \frac{44}{3} & 8 + 4 - \frac{34}{3} \end{bmatrix} \\
&= \begin{bmatrix} -\frac{19}{3} & \frac{5}{3} \\ -\frac{25}{6} & \frac{17}{6} \\ \frac{41}{3} & \frac{2}{3} \end{bmatrix}
\end{aligned}$$

bi) $x + 2y + z = 18$

$3x + 2y + 2z = 24$

$3x + 4y + z = 36$

$$\text{bii)} \quad \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 24 \\ 36 \end{bmatrix}$$

$$\text{biii)} \quad AX = B$$

$$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} -1 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \\ 1 & \frac{1}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 18 \\ 24 \\ 36 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(18) + \left(\frac{1}{3}\right)(24) + \left(\frac{1}{3}\right)(36) \\ \left(\frac{1}{2}\right)(18) + \left(-\frac{1}{3}\right)(24) + \left(\frac{1}{6}\right)(36) \\ (1)(18) + \left(\frac{1}{3}\right)(24) + \left(-\frac{2}{3}\right)(36) \end{bmatrix}$$

$$= \begin{bmatrix} -18 + 8 + 12 \\ 9 - 8 + 6 \\ 18 + 8 - 24 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 7 \\ 2 \end{bmatrix}$$

Exam pad = RM2, Pen = RM7, Liquid paper = RM2

$$\text{biv)} \quad x - \text{Exampad}; \quad y - \text{Pen}; \quad z - \text{Liquid Paper}$$

$$\text{Total price} = 4(2) + 5(7) + 1(2)$$

$$= 8 + 35 + 2$$

$$= \text{RM45.00}$$