QM016/1
Mathematics
Paper 1
Semester I
2006/2007
2 hours

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Matematik
Kertas 1
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BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **13** halaman bercetak.
This booklet consists of **13** *printed pages.*

KANG KOOI WEI

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks allocated for each question or section is shown in the bracket at the end of each question or section.

All steps must be shown clearly.

Only non-programmable scientific calculator can be used.

Numerical answers can be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Arithmetic Series:

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Geometric Series:

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{for } r < 1$$

Binomial Expansion:

$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n$, where $n \in \mathrm{N}$ and $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots \quad \text{for } |x| < 1$$

1. If $P = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 0 & -1 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$, find matrix R such that

$$R + 2(PQ) = \begin{bmatrix} 0 & 2 & 2 \\ -2 & 4 & 3 \\ -4 & 5 & 3 \end{bmatrix}.$$

[5 marks]

2. By substituting $a = 2^x$, solve the equation

$$4^x + 3 = 2^{x+2}.$$

[6 marks]

3. Obtain the solution set for

$$|2x+1| > -x^2 + 4 .$$

[7 marks]

4. The sum of the first k terms of an arithmetic series is 777. The first term is -3 and the k-th term is 77 . Obtain the value of k and the eleventh term of the series.

[7 marks]

5. (a) Find the values of A, B, C and D for the expression $\dfrac{4x^3 - 3x^2 + 6x - 27}{x^4 + 9x^2}$ in the form of partial fractions $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+9}$ where A, B, C and D are constants.

[5 marks]

(b) Given $A = \begin{bmatrix} -2 & 0 & 0 \\ -4 & 6 & -2 \\ 6 & -4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 0 \\ -1 & \frac{1}{5} & -\frac{1}{5} \\ -1 & -\frac{2}{5} & -\frac{3}{5} \end{bmatrix}$. Show that $AB = kI$ where k is a constant and I is an identity matrix. Find the value of k and hence obtain A^{-1}.

[5 marks]

KANG KOOI WEI

6. (a) Given the complex number z and its conjugate $\bar{z}$ satisfy the equation $z\bar{z} + 2z = 12 + 6i$. Find the possible values of z.

[6 marks]

(b) An equation in a complex number system is given by

$$z = \frac{1}{(z_1 - z_2)} + \frac{1}{\bar{z}_1}$$

where $z_1 = 1 + 2i$ and $z_2 = 2 - i$. Find

(i) the value of z in the Cartesian form $a + ib$

[3 marks]

(ii) the modulus and argument of z. [3 marks]

7. (a) Find the solution set of the inequality

$$\frac{1}{3 - 2x} < \frac{1}{x + 4}.$$ [5 marks]

(b) Solve the following inequality equation for all x is real numbers. Write your answer in set form.

$$4 - \left|\frac{3 - 2x}{1 + x}\right| \geq 1$$

[7 marks]

8. (a) Show that $(x - 3)$ is a factor of the polynomial

$$P(x) = x^3 - 2x^2 - 5x + 6.$$

Hence, factorize $P(x)$ completely.

[4 marks]

(b) If $f(x) = ax^2 + bx + c$ leaves remainder 1, 25 and 1 on division by $(x - 1)$, $(x + 1)$ and $(x - 2)$ respectively, find the values of a, b and c. Hence, show that $f(x)$ has two equal real roots.

[9 marks]

9. (a) Find the first four terms in the binomial expansion of the following functions:

(i) $\sqrt{1+2x}$ [2 marks]

(ii) $\dfrac{1}{(1-x)^2}$. [2 marks]

(b) Hence, expand $\sqrt{\dfrac{1+2x}{(1-x)^4}}$ in ascending power of x up to the term containing x^3. By putting $x=\dfrac{1}{10}$, show that $\sqrt{12000}$ is approximately $\dfrac{10935}{100}$.

[9 marks]

10. A doctor prescribed to a patient 13 units of vitamin A, 22 units of vitamin D and 31 units of vitamin E each day. The patient can choose from the combination of three brands of capsules; L, M and N. Each capsule of brand L contains 1 unit each of vitamins A, D and E. Each capsule of brand M contains 1 unit of vitamins A, 2 units of vitamin D, and 3 units of vitamin E. Each capsule of brand N contains 4 units of vitamins A, 7 units of vitamin D and 10 units of vitamin E. The above information is summarized in the following table:

Type of Vitamins	**Brand of Capsules**			**Total Unit of Vitamins**
	L	**M**	**N**	
A	1	1	4	13
D	1	2	7	22
E	1	3	10	31

By using x as the number of capsules of brand L, y the number of capsules of brand M and z the number of capsules of brand N,

(a) form a system of linear equations from the above information.

[2 marks]

(b) write the above system of linear equations in the form of matrix equation: $AX = B$, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix. Solve the system of equations by using the Gauss-Jordan elimination method.

[8 marks]

(c) determine the possible combinations of the number of capsules of brand L, M and N to be taken each day.

[3 marks]

(d) If brand L costs 10 cents per capsule, brand M costs 30 cents per capsule and brand N costs 60 cents per capsule. Determine the combination that will minimize the patient's daily cost.

[2 marks]

END OF QUESTION BOOKLET