

QM016/2
Mathematics
Paper 2
Semester I
Session 2007/2008
2 hours

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Matematik
Kertas 2
Semester I
Sesi 2007/2008
2 jam



BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK

Kertas 2

2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **11** halaman bercetak.

This booklet consists of 11 printed pages.

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks allocated for each question or section is shown in the bracket at the end of each question or section.

All steps must be shown clearly.

Only non-programmable scientific calculator can be used.

Numerical answers can be given in the form of π , e , surd, fractions or correct to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Differentiation

If $y = g(t)$ and $x = f(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}}$$

Integration

$$\int u dv = uv - \int v du$$

1. A function f is defined by $f(x) = x^2 - 2x - 3$ for $0 \leq x \leq 5$. State the range of f and determine whether f is one to one. [6 marks]

2. If $y^3 = \ln(x^3 y^2)$ for $x > 0, y > 0$, then find $\frac{dy}{dx}$ when $y = 1$. [6 marks]

3. Let $y = x(\ln x)^2, x > 0$. Show that

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 2x. \quad [6 \text{ marks}]$$

4. Given $h(x) = \frac{3x}{x-3}$. Defining $h^2(x) = (h \circ h)(x)$, determine the function $h^2(x)$ and hence deduce the inverse of $h(x)$. Evaluate $h^{13}(9)$.

[7 marks]

5. Given $\frac{2x^3 + 9x^2 + 4x - 7}{2x^2 + 9x + 4} = g(x) + \frac{A}{2x+1} + \frac{B}{x+4}$. Determine the function $g(x)$ and

find the values of A and B . Hence, find $\int \frac{2x^3 + 9x^2 + 4x - 7}{2x^2 + 9x + 4} dx$.

[10 marks]

6. The function f is defined as

$$f(x) = \begin{cases} \frac{x^2 + x - 12}{|x - 3|}, & x < 3 \\ A, & x = 3 \\ 2x - B, & 3 < x \leq 4 \\ C, & x > 4. \end{cases}$$

- (a) Find $\lim_{x \rightarrow 3^-} f(x)$ and $\lim_{x \rightarrow 3^+} f(x)$. [5 marks]
- (b) Use the definition of continuity to determine the values of A and B if f is continuous at $x = 3$. [3 marks]
- (c) For what values of C is f discontinuous at $x = 4$? [4 marks]

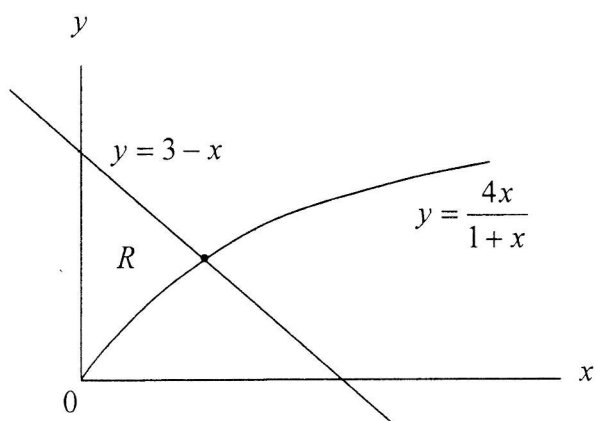
7. Given $f(x) = 2x^2 + 1$, $x \geq 0$ and $g(x) = x - 3$, find
- (a) the inverse of f and g and verify that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$, [6 marks]
- (b) the function h if $(g \circ f)^{-1} \circ h(x) = \frac{1}{x}$, [3 marks]
- (c) the values of x for which $f \circ g = g \circ f$. [3 marks]

8. Given $f(x) = \frac{2x - 3}{(x - 1)(x + 3)}$. Find
- (a) the domain of f , [1 mark]
- (b) the x -intercept and y -intercept of f , [2 marks]
- (c) the vertical asymptote(s) of f , [2 marks]
- (d) $\lim_{x \rightarrow -\infty} f(x)$ and $\lim_{x \rightarrow +\infty} f(x)$. Hence, state the horizontal asymptote of f . [4 marks]
- Sketch the graph of f . [4 marks]

9. (a) Find $\frac{dy}{dx}$ when $x = 0$ for each of the following:
- (i) $y = \ln(x + \sqrt{x^2 + 1})$, [3 marks]
- (ii) $y = \frac{e^{2x}(2x^3 + 1)}{\sqrt{x + 1}}$. [4 marks]
- (b) Given $x = 3t - \frac{2}{t}$, $y = 2t + \frac{3}{t}$, $t \neq 0$. Show that
- $$\frac{dy}{dx} = \frac{2}{3} - \frac{1}{3} \left(\frac{13}{3t^2 + 2} \right).$$
- Hence find $\frac{d^2y}{dx^2}$. [6 marks]

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10.



In the figure above, R is the region bounded by the line $y = 3 - x$, the curve $y = \frac{4x}{1+x}$ and the y -axis. Find

- (a) the area of R , [7 marks]
- (b) the volume of solid obtained when R is rotated through 360° about the x -axis.
Give your answer in term of π . [8 marks]

END OF QUESTION BOOKLET