

QM016/1
Mathematics
Paper 1
Semester I
2007/2008
2 hours



QM016/1
Matematik
Kertas 1
Semester I
2007/2008
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **11** halaman bercetak.
This booklet consists of 11 printed pages.

© Bahagian Matrikulasi

QM016/1

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks allocated for each question or section is shown in the bracket at the end of each question or section.

All steps must be shown clearly.

Only non-programmable scientific calculator can be used.

Numerical answers can be given in the form of π , e , surd, fractions or correct to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Arithmetic Series:

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Geometric Series:

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad \text{for } r < 1$$

Binomial Expansions:

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n, \text{ where } n \in \mathbb{N} \text{ and}$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots \text{ for } |x| < 1$$

1. Given that $81^y = 3^{(2y-3)x}$ and $2^{18y+6x} = 64^{xy}$. Find the values of x and y .
[6 marks]
2. Express $\frac{2x+1}{(x+2)(x^2-2x+4)}$ in partial fractions. [6 marks]
3. If $z_1 = 4 - i$ and $z_2 = 1 - 2i$, find $z_1 - \frac{5}{z_2}$. Express the answer in polar form.
[6 marks]
4. The sum of the first n terms of an arithmetic series is $\frac{n}{2}(3n-5)$. If the second and fourth terms of the arithmetic series are the second and the third terms of a geometric series respectively, find the sum of the first eleven terms of this geometric series.
[7 marks]
5. The quadratic equation $x^2 + k(x+2) - (x+6) = 0$ has roots α and β , where k is a constant.
- (a) Find a quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ in terms of k .
[5 marks]
- (b) Find $\alpha^2 + \beta^2$ in terms of k . Hence, determine the minimum value of $\alpha^2 + \beta^2$.
[4 marks]
6. (a) Find a cubic polynomial $Q(x) = (x+a)(x+b)(x+c)$ satisfying the following conditions:
the coefficients of x^3 is 1, $Q(-1) = 0$, $Q(2) = 0$, and $Q(3) = -8$.
[4 marks]
- (b) A polynomial $P(x) = ax^3 - 4x^2 + bx + 18$ has a factor $(x+2)$ and a remainder $(2x+18)$ when divided by $(x+1)$. Find the values of a and b . Hence, factorize $P(x)$ completely.
[8 marks]

7. Solve the following inequalities:

(a) $\frac{x}{x+4} \leq \frac{1}{2x-1}$. [6 marks]

(b) $\left| \frac{x}{x+4} \right| < 2$. [7 marks]

8. A system of linear equations is given as

$$\begin{aligned} ax - 2y - 3z &= b \\ 2x - y + 4z &= 2 \\ 4x + 3y - 2z &= 14 \end{aligned}$$

where a and b are constants.

(a) Find x and z in terms of a and b using Cramer's rule. [9 marks]

(b) Determine the conditions of a and b for which the above system

(i) has a unique solution.

(ii) has no solution.

[4 marks]

9. Given that $A = \begin{bmatrix} 1 & a & 2 \\ 2 & 1 & 2 \\ 2 & 2 & b \end{bmatrix}$, where a and b are constants.

(a) If $|A| = -13$, evaluate the determinant of matrix $\begin{bmatrix} 4 & 1 & 2 \\ 2 & a & 2 \\ 4 & 2 & b \end{bmatrix}$ using determinant properties.

[4 marks]

(b) Given that $A^2 - 4A = 5I$, where I is a 3×3 identity matrix. Show that $a = 2$ and $b = 1$. Hence, find A^{-1} .

[9 marks]

QM016/1

10. Given that $f(x) = \frac{1}{1+x}$, $x \neq -1$ and $g(x) = \frac{1}{2-x}$, $x \neq 2$.

(a) Expand $f(x)$ and $g(x)$ as a series of ascending powers of x up to the term containing x^n . Hence, estimate the value of $(1.9)^{-1}$ using the first four terms of $g(x)$.

[7 marks]

(b) If $h(x) = f(x) + g(x)$, show that the coefficient of x^n for $h(x)$ is $(-1)^n + \frac{1}{2^{n+1}}$. Hence, obtain the coefficient of x^3 for $h(x)$.

[5 marks]

(c) Find the coefficient of x^2 for $\frac{f(x)}{g(x)}$.

[3 marks]

END OF QUESTION BOOKLET