

QM016/2
Mathematics
Paper 2
Semester I
Session 2008/2009
2 hours

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Matematik
Kertas 2
Semester I
Sesi 2008/2009
2 jam



BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **11** halaman bercetak.

This booklet consists of 11 printed pages.

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks allocated for each question or section is shown in the bracket at the end of each question or section.

All steps must be shown clearly.

Only non-programmable scientific calculator can be used.

Numerical answers can be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Differentiation

If $y = g(t)$ and $x = f(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Integration

$$\int u dv = uv - \int v du$$

1. Given $\ln y = e^{xy}$, find $\frac{dy}{dx}$.

[5 marks]

2. If $y = \sqrt{2x^2 + 5x - 3}$, determine the domain of $\frac{dy}{dx}$ and find the respective intervals in which $\frac{dy}{dx} > 0$ and $\frac{dy}{dx} < 0$.

[6 marks]

3. Given that $f(x) = \frac{10 - 2x}{k}$ and $g(x) = 5 - 2x^2$. Find the value of k so that $f^{-1}(x^2) = g\left(\frac{x}{2}\right)$. Hence, find $(f^{-1} \circ g)(0)$.

[7 marks]

4. Let $f(x) = |4x - 1|$ and $g(x) = x + 2$.

(a) Find the interval of x for which $f(x) < g(x)$.

[4 marks]

(b) If $h(x) = f(x) + 2g(x)$, express $h(x)$ as a piecewise function.

[3 marks]

5. Let $f(ax) = a^3x^2 + a^2x + 3a$ where a is non-zero.

(a) Find a if $f(0) = 6$.

[2 marks]

(b) Determine $f(x)$.

[3 marks]

(c) Determine the domain and range of $f(x)$. Hence, state the interval in which f is one to one.

[5 marks]

6. (a) By using the partial fraction method, show that

$$\frac{1}{x^2 - 4} = \frac{1}{4} \left(\frac{1}{x-2} - \frac{1}{x+2} \right).$$

Hence, find $\int \frac{x^2 + 1}{x^2 - 4} dx$.

[6 marks]

- (b) Sketch the region bounded by the curves $y = xe^{x^2}$, $y = x^2$, $x \geq 0$ and the line $x = 2$. Find its area.

[6 marks]

7. Given

$$f(x) = \begin{cases} e^x + A, & x < 0 \\ x^2 - 2x + 3, & 0 \leq x < 1 \\ x + B, & x \geq 1. \end{cases}$$

- (a) Determine the values of A and B for f to be continuous.

[4 marks]

- (b) Find the minimum value of f .

[3 marks]

- (c) Is f differentiable? Justify your answer by using the first principle of differentiation.

[Hint: $e^x = 1 + x + \frac{x^2}{2!} + \dots$]

[5 marks]

8. Given that

$$y = e^t + e^{-t} \quad \text{and} \quad x = e^{-t}.$$

- (a) Find the point (x, y) on the curve where $\frac{dy}{dx} = 0$.

[6 marks]

- (b) Solve for t if

$$\left(\frac{d^2y}{dx^2} \right)^2 + \frac{dy}{dx} - 1 = 0.$$

[7 marks]

9. Evaluate

(a) $\int \frac{1}{1+e^{-x}} dx$.

[7 marks]

(b) $\int \ln(x^x) dx$.

[6 marks]

10. Given $f(x) = \frac{x|x-1|}{(x-1)(x+2)}$.

(a) Show that f is equivalent to

$$g(x) = \begin{cases} \frac{x}{x+2}, & x > 1 \\ -\frac{x}{x+2}, & x < 1 \end{cases}$$

[3 marks]

(b) Determine the asymptotes and the points of discontinuity of g .

[6 marks]

(c) Sketch the graph of g .

[3 marks]

(d) Find the points of intersection of $g(x)$ with the straight line $y = x + 2$.

[3 marks]

END OF BOOKLET