QM016/2
Mathematics
Paper 2
Semester I
2009/2010
2 hours

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Matematik
Kertas 2
Semester I
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2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **11** halaman bercetak.
This booklet consists of **11** *printed pages.*

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Differentiation

If $y = g(t)$ and $x = f(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Integration

$$\int udv = uv - \int vdu$$

1 A function g is defined by

$$g(x) = \frac{1}{\sqrt{x-1}}, \quad x > 1.$$

Find $g^{-1}(x)$ and state its domain and range.

[5 *marks*]

2 A function f is given as

$$f(x) = \begin{cases} |x+1|, & x < 0 \\ 2, & x = 0 \\ e^{-2x}, & x > 0. \end{cases}$$

Find $\lim_{x \to 0^-} f(x)$, $\lim_{x \to 0^+} f(x)$ and $\lim_{x \to 0} f(x)$.

Hence, determine whether f is continuous at $x = 0$. Give a reason to your answer.

[6 *marks*]

3 If $y = x + e^x$, show that $\frac{d^2x}{dy^2} + \frac{e^x}{(1+e^x)^3} = 0$.

[6 *marks*]

4 Evaluate $\int_2^3 \frac{x+1}{x^2(x-1)}\, dx$.

[7 *marks*]

5 A parametric curve is given by $x = t - \frac{1}{t}$, $y = t + \frac{1}{t}$, $t \neq 0$.

(a) Find $\frac{dy}{dx}$ in terms of t and evaluate it at $t = -2$.

[4 *marks*]

(b) Find the value of $\frac{d^2y}{dx^2}$ at $t = 1$, and evaluate $\lim_{t \to \infty} \frac{d^2y}{dx^2}$.

[8 *marks*]

6 (a) Show that $y-\sqrt{y^2+1}<0$ for all real values of y.

[2 *marks*]

(b) Let f be a function defined by $f(x)=\dfrac{e^x-e^{-x}}{2}$. Find $f^{-1}(x)$.

[6 *marks*]

(c) Evaluate $\lim\limits_{x\to 0}\dfrac{e^{2x}-e^{-2x}}{e^x-e^{-x}}$.

[3 *marks*]

7 A function f is defined by

$$f(x)=\begin{cases} 34, & x=-4 \\ 0, & x=2 \\ 17, & x=4 \\ \dfrac{x^4-3x^2-4}{x^2+x-6}, & x\neq -4, x\neq -3, x\neq 2, x\neq 4 \end{cases}$$

(a) Evaluate $\lim\limits_{x\to 2} f(x)$.

[4 *marks*]

(b) Find the interval(s) where f is continuous on the interval $[-4,4]$.

[8 *marks*]

8 (a) Given a function g defined by

$$g(x)=\begin{cases} xe^{x^2}, & x\leq 1 \\ \dfrac{(\ln x)^2}{x}, & x>1. \end{cases}$$

Evaluate $\int_{-1}^{3} g(x)\,dx$.

[6 *marks*]

(b) Use integration by parts to show that

$$\int\frac{xe^{2x}}{\sqrt{e^{2x}+1}}\,dx=(x-1)\sqrt{e^{2x}+1}-\int\frac{1}{\sqrt{e^{2x}+1}}\,dx.$$

[7 *marks*]

9 (a) Let f and g be functions such that $f(x) = x^2 g(x^2)$ with $g(1) = 2$ and $g'(1) = 1$. Find $f'(1)$.

[4 *marks*]

(b) Given a curve $y = x + \frac{1}{x}$.

(i) Determine the gradient of the curve $y = x + \frac{1}{x}$ at $x = b$ in terms of b.

(ii) Find the value of b if a straight line with the gradient in (i) passes through the points $(b, b + \frac{1}{b})$ and $(0, 4)$.

(iii) Hence, find the equation of a line perpendicular to the line in (ii) at $(0, 4)$.

[9 *marks*]

10 A region R is bounded by the curve $y = x(x - 2)$ and line $y = x$.

(a) Sketch the graphs and shade the region R.

[2 *marks*]

(b) Find the area of R.

[3 *marks*]

(c) Find the volume of the solid obtained when the part of R above the x-axis is rotated through $360°$ about the x-axis.

[5 *marks*]

(d) Let R forms the surface of water in a pond where the depth of the water at any point (x, y) in R is given by $x + 5$. Find the volume of the water in the pond.

[5 *marks*]

END OF QUESTION BOOKLET