

QS016/2
Mathematics
Paper 2
Semester I
Session 2010/2011
2 hours

QS016/2
Matematik
Kertas 2
Semester I
Sesi 2010/2011
2 jam



BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **15** halaman bercetak.
This booklet consists of 15 printed pages.

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Differentiation

If $y = g(t)$ and $x = f(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}}$$

Integration

$$\int u dv = uv - \int v du$$

1 Find $\frac{dy}{dx}$ for each of the following:

(a) $y = (\ln x)^5$.

[2 marks]

(b) $xy^2 - ye^x = 3$.

[4 marks]

2 Find the exact value of $\int_1^{\sqrt{2}} t^3 \sqrt{t^2 - 1} dt$.

[6 marks]

3 If f is a function with $f'(1) = 2$, find $\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{\sqrt{x} - 1}$.

[6 marks]

4 Express $\frac{8x^2 + 15}{2x^3 + 3x}$ as partial fractions.

Hence, evaluate $\int \frac{8x^2 + 15}{2x^3 + 3x} dx$.

[7 marks]

5 Given the functions f and g as follows:

$$f(x) = 2 - x^2,$$

$$g(x) = x + 2.$$

(a) Find $f \circ g$ and $g \circ f$.

[4 marks]

(b) State the domain and range of $f \circ g$.

[3 marks]

(c) Find $(g \circ f)^{-1}$.

[2 marks]

(d) Determine the value of x such that $f \circ g(x) = g \circ f(x)$.

[3 marks]

6 (a) State the conditions of continuity of a function at a point $x = c$.

[2 marks]

(b) A function f defined by

$$f(x) = \begin{cases} \frac{|x-2|}{x^2+3x-10}, & -5 < x < 2 \\ A, & 2 \leq x < 3 \\ Ax+B, & x = 3 \end{cases}$$

is continuous at $x = 2$ and $x = 3$.

(i) Find $\lim_{x \rightarrow 2^-} f(x)$.

[6 marks]

(ii) Determine the values of the constants A and B .

[5 marks]

7 (a) Evaluate.

$$(i) \lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + 2x - 1}}{x + 1}.$$

[3 marks]

$$(ii) \lim_{x \rightarrow -3} \frac{2 - \sqrt{x^2 - 5}}{x + 3}.$$

[4 marks]

(b) If $\lim_{x \rightarrow 4} \frac{f(x) - 5}{x - 2} = 1$, find $\lim_{x \rightarrow 4} f(x)$.

[3 marks]

8 Consider the curve given by the equation $f(x) = 2 - x^2$.

(a) Sketch the region bounded by the curves $f(x)$, $g(x) = x^2$, the lines $x = 0$ and $x = 2$. Hence, find the area of the region.

[7 marks]

(b) Find the volume of solid generated when the region bounded by the curve $f(x)$, lines $x = 1$ and $x = 2$ is rotated completely about the x -axis.

[5 marks]

9 Consider the parametric equations

$$x = 2t - t^{-1}, \quad y = 2t + t^{-1}, \quad t \geq 1.$$

(a) Show that

$$\frac{dy}{dx} = \frac{2t^2 - 1}{2t^2 + 1}.$$

[3 marks]

(b) Evaluate $\frac{dy}{dx}$ at the point (1, 3).

[4 marks]

(c) Find $\frac{d^2y}{dx^2}$ in term of t . Hence, show that

$$\frac{d^2y}{dx^2} = \frac{8}{y^3}.$$

[6 marks]

10 A function f is defined by $f(x) = \frac{5x^2 + 8x + 4}{x^2 + x}$.

(a) Find the vertical and horizontal asymptotes of f .

[3 marks]

(b) Find the coordinates of the point where the curve f cuts the horizontal asymptote.

[2 marks]

(c) Determine the coordinates of the point where $f'(x) = 0$.

[3 marks]

(d) By writing $y = f(x)$, show that

$$(y - 5)x^2 + (y - 8)x - 4 = 0$$

Hence, for real x , show that $f(x) \leq -4$ or $f(x) \geq 4$.

[4 marks]

(e) Sketch the graph of f .

[3 marks]

END OF QUESTION BOOKLET