KEMENTERIAN PENDIDIKAN MALAYSIA

QS 015/1

Matriculation Programme Examination

Semester I

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1. Evaluate the solution of $4^{y-2} = \frac{1}{3^{-y}}$ up to three decimal places.

2. The first three terms of a geometric sequence are $\left(\frac{4}{3}m - 2\right), (2\text{m} - 1)$ and 12.

   Determine the value of m. Hence, find the sixth term for this sequence.

3. Solve the equation

   $$2 + \log_2 x = 15 \log_{\text{x}} 2\,.$$

4. (a) Determine the values of x so that $\begin{bmatrix} 1 & x & -1 \\ x & 0 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is singular.

   (b) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$, find C when $A = BCB^{-1}$.

5. (a) Expand $(2 + x)^{-\frac{1}{2}}$ in ascending powers of x, up to the term x^3.

   (b) Use the expansion in (a) to approximate $\sqrt{\frac{2}{3}}$.

6. Given $z_1 = 3 - 3\text{i}$ and $z_2 = 3 + 2\text{i}$.

   (a) Write $\overline{z_1}$ in polar form.

   (b) Express $\frac{(\overline{z_1} z_2)}{13} + \overline{\left[\frac{\iota^3}{-z_2}\right]}$ in the form of $a + bi, c, b \in R$.

7. A curve $y = ax^2 + \text{bx} + \text{c}$ where a, b and c are constants, passes through the points (2,11), (-1, -16) and (3,28).

   (a) By using the above information, construct a system containing three linear equations.

(b) Express the above system as a matrix equation AX=B.

(c) Find the inverse of matrix A by using the adjoint matrix method. Hence, obtain the values of a, b and c.

8. Given a function $f(x) = \sqrt{3-2x}$.

(a) Show that f is a one to one function.

(b) Find the domain and range of f.

(c) Determine the inverse function of f and state its domain and range.

(d) Sketch the graphs of f and f^{-1} on the same axis.

9. (a) The function f is given as $f(x) = \frac{ax+2}{3x-4}, \mathrm{x} \neq \frac{4}{3}$. If $(f\ o\ f)(\mathrm{x}) = \mathrm{x}$, find the value of a.

(b) Let $f(x) = ln|3x+2|$ and $g(x) = e^{-x} + 2$ be two functions. Evaluate $(g\ o\ f)^{-1}(3)$.

10. (a) Solve the inequality $\left|\frac{x-1}{x+3}\right| > 2$.

(b) Show that $\frac{2^x\ x\ 4^{2x}}{8^x} = 2^{2x}$.

Hence, find the interval for x so that $\frac{2^x \text{ x } 4^{2x}}{8^x} - 13(2^x) + 36 \geq 0$.

END OF QUESTION PAPER

1. Evaluate the solution of $4^{y-2} = \frac{1}{3^{-y}}$ up to three decimal places.

SOLUTION

$$4^{y-2} = \frac{1}{3^{-y}}$$

$$4^{y-2} = 3^y$$

$$\ln 4^{y-2} = \ln 3^y$$

$$(y-2)\ln 4 = y \ln 3$$

$$y \ln 4 - 2\ln 4 = y \ln 3$$

$$y \ln 4 - y \ln 3 = 2\ln 4$$

$$y(\ln 4 - \ln 3) = 2\ln 4$$

$$y = \frac{2\ln 4}{(\ln 4 - \ln 3)}$$

$$= 9.638$$

2. The first three terms of a geometric sequence are $\left(\frac{4}{3}m-2\right)$, $(2m-1)$ and 12.

Determine the value of m. Hence, find the sixth term for this sequence.

SOLUTION

Geometric Sequence *If* $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ *are three consecutive terms,* $b = \pm\sqrt{ac}$ $\left(\frac{4}{3}m-2\right), (2m-1), 12$ $(2m-1) = \sqrt{\left(\frac{4}{3}m-2\right)(12)}$ $(2m-1) = \sqrt{16m-24}$ $(2m-1)^2 = 16m-24$ $4m^2-4m+1 = 16m-24$ $4m^2-20m+25 = 0$ $(2m-5)(2m-5) = 0$ $m = \frac{5}{2}$	**Alternative** $\boldsymbol{r} = \frac{\boldsymbol{T_2}}{\boldsymbol{T_1}} = \frac{\boldsymbol{T_3}}{\boldsymbol{T_2}}$ $\frac{2m-1}{\frac{4}{3}m-2} = \frac{12}{2m-1}$ $(2m-1)^2 = 16m-24$ $4m^2-20m+25 = 0$ $(2m-5)(2m-5) = 0$ $m = \frac{5}{2}$

When $m = \frac{5}{2}$,

$$T_1 = a = \frac{4}{3}m - 2 = \frac{4}{3}\left(\frac{5}{2}\right) - 2 = \frac{4}{3}$$

$$T_2 = 2m - 1 = 2\left(\frac{5}{2}\right) - 1 = 4$$

$$\boldsymbol{r = \frac{T_2}{T_1}}$$

$$r = \frac{4}{\frac{4}{3}} = 3$$

$$\boldsymbol{T_n = ar^{n-1}}$$

$$T_6 = \left(\frac{4}{3}\right)(3)^5$$

$$= 324$$

3. Solve the equation

$$2 + \log_2 x = 15 \log_x 2.$$

SOLUTION

$$2 + \log_2 x = 15 \log_x 2$$

$$2 + \log_2 x = 15 \left(\frac{\log_2 2}{\log_2 x}\right)$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$2 + \log_2 x = 15 \left(\frac{1}{\log_2 x}\right)$$

$$2 + \log_2 x = \frac{15}{\log_2 x}$$

$$Let\ u = \log_2 x$$

$$2 + u = \frac{15}{u}$$

$$2u + u^2 = 15$$

$$u^2 + 2u - 15 = 0$$

$$(u - 3)(u + 5) = 0$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$u = 3$	$u = -5$
$\log_2 x = 3$	$\log_2 x = -5$
$x = 2^3$	$x = 2^{-5}$
$x = 8$	$x = \frac{1}{32}$

4. (a) Determine the values of x so that $\begin{bmatrix} 1 & x & -1 \\ x & 0 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is singular.

(b) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$, find C when $A = BCB^{-1}$.

SOLUTION

(a)

Singular Matrix

Singular matrix is square matrix whose determinant is zero.

$$\begin{vmatrix} 1 & x & -1 \\ x & 0 & 1 \\ 1 & 3 & -1 \end{vmatrix} = 0$$

$$(1)\begin{vmatrix} 0 & 1 \\ 3 & -1 \end{vmatrix} - (x)\begin{vmatrix} x & 1 \\ 1 & -1 \end{vmatrix} + (-1)\begin{vmatrix} x & 0 \\ 1 & 3 \end{vmatrix} = 0$$

$$(1)[0-3] - (x)[-x-1] + (-1)[3x-0] = 0$$

$$-3 + x^2 + x - 3x = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3 \qquad x = -1$$

(b)

$$A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

$$A = BCB^{-1}$$

$$B^{-1}(A) = B^{-1}(BCB^{-1})$$

$$B^{-1}(A) = ICB^{-1}$$

$$B^{-1}A = CB^{-1}$$

$$(B^{-1}A)B = (CB^{-1})B$$

$$B^{-1}AB = CI$$

$$C = B^{-1}AB$$

$$A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{-1-0}\begin{bmatrix} -1 & -4 \\ 0 & 1 \end{bmatrix}$$

$$= -1\begin{bmatrix} -1 & -4 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

$$C = B^{-1}AB$$

$$= \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 13 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 15 \\ -1 & -2 \end{bmatrix}$$

5. (a) Expand $(2+x)^{-\frac{1}{2}}$ in ascending powers of x, up to the term x^3.

(b) Use the expansion in (a) to approximate $\sqrt{\frac{2}{3}}$.

SOLUTION

(a)

$$(2+x)^{-\frac{1}{2}} = \left[2\left(1+\frac{x}{2}\right)\right]^{-\frac{1}{2}}$$

$$= 2^{-\frac{1}{2}}\left(1+\frac{x}{2}\right)^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2}}\left[1+\frac{\left(-\frac{1}{2}\right)}{1!}\left(\frac{x}{2}\right)^1+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(\frac{x}{2}\right)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(\frac{x}{2}\right)^3+\cdots\right]$$

$$= \frac{1}{\sqrt{2}}\left[1-\frac{x}{4}+\frac{3}{8}\left(\frac{x^2}{4}\right)-\frac{15}{48}\left(\frac{x^3}{8}\right)+\cdots\right]$$

$$= \frac{1}{\sqrt{2}}\left[1-\frac{x}{4}+\frac{3x^2}{32}-\frac{5x^3}{128}+\cdots\right]$$

$$= \frac{1}{\sqrt{2}}-\frac{1}{4\sqrt{2}}x+\frac{3}{32\sqrt{2}}x^2-\frac{5}{128\sqrt{2}}x^3+\cdots$$

(b)

$$\sqrt{\frac{2}{3}} = \left(\frac{2}{3}\right)^{\frac{1}{2}}$$

$$= \left(\frac{3}{2}\right)^{-\frac{1}{2}}$$

$$(2+x)^{-\frac{1}{2}} = \left(\frac{3}{2}\right)^{-\frac{1}{2}}$$

$$2+x = \frac{3}{2}$$

$$x = -\frac{1}{2}$$

$$(2+x)^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}}x + \frac{3}{32\sqrt{2}}x^2 - \frac{5}{128\sqrt{2}}x^3$$

$$\left[2+\left(-\frac{1}{2}\right)\right]^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}}\left(-\frac{1}{2}\right) + \frac{3}{32\sqrt{2}}\left(-\frac{1}{2}\right)^2 - \frac{5}{128\sqrt{2}}\left(-\frac{1}{2}\right)^3$$

$$\left(\frac{3}{2}\right)^{-\frac{1}{2}} = 0.8155$$

$$\left(\frac{2}{3}\right)^{\frac{1}{2}} = 0.8155$$

$$\sqrt{\frac{2}{3}} = 0.8155$$

6. Given $z_1 = 3 - 3\text{i}$ and $z_2 = 3 + 2\text{i}$.

(a) Write $\overline{z_1}$ in polar form.

(b) Express $\frac{(\overline{z_1} z_2)}{13} + \left[\overline{\frac{\iota^3}{-z_2}}\right]$ in the form of $a + bi, c, b \in R$.

SOLUTION

(a)

$z_1 = 3 - 3\text{i}$ and $z_2 = 3 + 2\text{i}$

$\overline{z_1} = 3 + 3i$

$r = |\overline{z_1}| = \sqrt{3^2 + 3^2}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$Argument\ of\ \overline{z_1}, \quad \theta = tan^{-1}\left(\frac{b}{a}\right)$

$= tan^{-1}\left(\frac{3}{3}\right)$

$= tan^{-1}(1)$

$= 0.785\ rad$

$\overline{z_1}$ in polar form

$\overline{z_1} = r[\cos\theta + isin\ \theta]$

$= 3\sqrt{2}[\cos 0.785 + isin\ 0.785]$

(b)

$$\frac{(\bar{z}_1 z_2)}{13} + \overline{\left(\frac{i^3}{-z_2}\right)} = \frac{(3+3i)(3+2i)}{13} + \overline{\left[\frac{-i}{-(3+2i)}\right]}$$

$$= \frac{9+6i+9i+6i^2}{13} + \overline{\left[\frac{i}{(3+2i)}\right]}$$

$$= \frac{3+15i}{13} + \overline{\left[\frac{i(3-2i)}{(3+2i)(3-2i)}\right]}$$

$$= \frac{3+15i}{13} + \overline{\left[\frac{3i+2}{9+4}\right]}$$

$$= \frac{3+15i}{13} + \overline{\left[\frac{2+3i}{13}\right]}$$

$$= \frac{3+15i}{13} + \frac{2-3i}{13}$$

$$= \frac{3+15i+2-3i}{13}$$

$$= \frac{5+12i}{13}$$

$$= \frac{5}{13} + \frac{12}{13}i$$

7. A curve $y = ax^2 + \mathrm{bx} + \mathrm{c}$ where a, b and c are constants, passes through the points (2,11), (-1, -16) and (3,28).

   (a) By using the above information, construct a system containing three linear equations.

   (b) Express the above system as a matrix equation AX=B.

   (c) Find the inverse of matrix A by using the adjoint matrix method. Hence, obtain the values of a, b and c.

<u>SOLUTION</u>

(a)

$y = ax^2 + \mathrm{bx} + \mathrm{c}$

$At\ (2,11) \rightarrow x = 2, y = 11$

$11 = a2^2 + \mathrm{b}(2) + \mathrm{c}$

$4a + 2\mathrm{b} + \mathrm{c} = 11$ .... (1)

$At\ (-1,-16) \rightarrow x = -1, y = -16$

$-16 = a(-1)^2 + \mathrm{b}(-1) + \mathrm{c}$

$a - \mathrm{b} + \mathrm{c} = -16$ .... (2)

$At\ (3,28) \rightarrow x = 3, y = 28$

$28 = a(3)^2 + \mathrm{b}(3) + \mathrm{c}$

$9a + 3\mathrm{b} + \mathrm{c} = 28$ .... (2)

(b)

$4a + 2b + c = 11$ .... (1)

$a - b + c = -16$ .... (2)

$9a + 3b + c = 28$ .... (2)

$$\begin{pmatrix} 4 & 2 & 1 \\ 1 & -1 & 1 \\ 9 & 3 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 11 \\ -16 \\ 28 \end{pmatrix}$$

(c)

$$A^{-1} = \frac{1}{|A|} Adj(A)$$

$$Adj(A) = (C)^T$$

$$A = \begin{pmatrix} 4 & 2 & 1 \\ 1 & -1 & 1 \\ 9 & 3 & 1 \end{pmatrix}$$

$$|A| = (4)\begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} - (2)\begin{vmatrix} 1 & 1 \\ 9 & 1 \end{vmatrix} + (1)\begin{vmatrix} 1 & -1 \\ 9 & 3 \end{vmatrix}$$

$$= 4[-1-3] - 2[1-9] + 1[3+9]$$

$$= -16 + 16 + 12$$

$$= 12$$

$\boldsymbol{Cofactor\ of\ A}$

$$C = \begin{bmatrix} +\begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 9 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & -1 \\ 9 & 3 \end{vmatrix} \\ -\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} & +\begin{vmatrix} 4 & 1 \\ 9 & 1 \end{vmatrix} & -\begin{vmatrix} 4 & 2 \\ 9 & 3 \end{vmatrix} \\ +\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 4 & 2 \\ 1 & -1 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -1-3 & -(1-9) & 3+9 \\ -(2-3) & 4-9 & -(12-18) \\ 2+1 & -(4-1) & -4-2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 8 & 12 \\ 1 & -5 & 6 \\ 3 & -3 & -6 \end{bmatrix}$$

Adjoin of A

$$Adj(A) = C^T$$

$$= \begin{bmatrix} -4 & 8 & 12 \\ 1 & -5 & 6 \\ 3 & -3 & -6 \end{bmatrix}^T$$

$$= \begin{bmatrix} -4 & 1 & 3 \\ 8 & -5 & -3 \\ 12 & 6 & -6 \end{bmatrix}$$

Inverse of A

$$\boldsymbol{A^{-1} = \frac{1}{|A|} Adj(A)}$$

$$A^{-1} = \frac{1}{12} \begin{bmatrix} -4 & 1 & 3 \\ 8 & -5 & -3 \\ 12 & 6 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{4}{12} & \frac{1}{12} & \frac{3}{12} \\ \frac{8}{12} & -\frac{5}{12} & -\frac{3}{12} \\ \frac{12}{12} & \frac{6}{12} & -\frac{6}{12} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{3} & \frac{1}{12} & \frac{1}{4} \\ \frac{2}{3} & -\frac{5}{12} & -\frac{1}{4} \\ 1 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\begin{pmatrix} 4 & 2 & 1 \\ 1 & -1 & 1 \\ 9 & 3 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 11 \\ -16 \\ 28 \end{pmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{12} & \frac{1}{4} \\ \frac{2}{3} & -\frac{5}{12} & -\frac{1}{4} \\ 1 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 11 \\ -16 \\ 28 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 7 \\ -11 \end{bmatrix}$$

$$\therefore a = 2, \quad b = 7, \quad c = -11$$

8. Given a function $f(x) = \sqrt{3 - 2x}$.

(a) Show that f is a one to one function.

(b) Find the domain and range of f.

(c) Determine the inverse function of f and state its domain and range.

(d) Sketch the graphs of f and f^{-1} on the same axis.

SOLUTION

$$f(x) = \sqrt{3 - 2x}$$

(a)

$$f(x_1) = \sqrt{3 - 2x_1}$$

$$f(x_2) = \sqrt{3 - 2x_2}$$

$$f(x_1) = f(x_2)$$

$$\sqrt{3 - 2x_1} = \sqrt{3 - 2x_2}$$

$$3 - 2x_1 = 3 - 2x_2$$

$$x_1 = x_2$$

$$\therefore f \text{ is a one to one function}$$

(b)

$$f(x) = \sqrt{3 - 2x}$$

$\boldsymbol{Domain\ of\ x}$:

D_f: $3 - 2x \geq 0$

$3 \geq 2x$

$2x \leq 3$

$x \leq \frac{3}{2}$

$\boldsymbol{D_f} = (-\infty, \frac{3}{2}]$

$\boldsymbol{R_f} = [\mathbf{0}, \infty)$

(c)

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$f(x) = \sqrt{3 - 2x}$

$y = \sqrt{3 - 2x}$

$y^2 = 3 - 2x$

$2x = 3 - y^2$

$x = \frac{3 - y^2}{2}$

$f^{-1}(x) = \frac{3 - x^2}{2}$

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<u>Alternative</u>

$f(x) = \sqrt{3 - 2x}$

$f[f^{-1}(x)] = x$

$\sqrt{3 - 2f^{-1}(x)} = x$

$3 - 2f^{-1}(x) = x^2$

$2f^{-1}(x) = 3 - x^2$

$f^{-1}(x) = \frac{3 - x^2}{2}$

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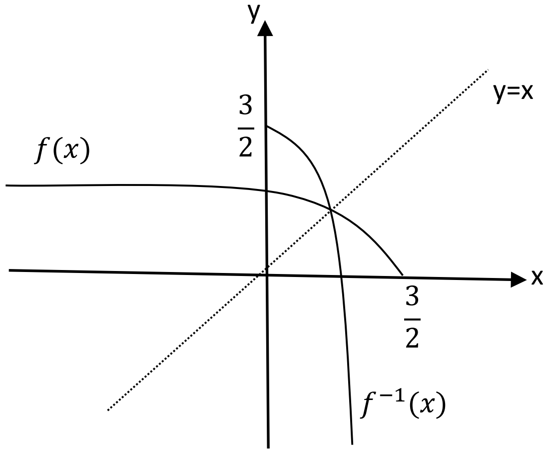
(d)



9. (a) The function f is given as $f(x)=\frac{ax+2}{3x-4}, \mathrm{x} \neq \frac{4}{3}$. If $(f\ o\ f)(\mathrm{x})=\mathrm{x}$, find the value of a.

(b) Let $f(x)=ln|3x+2|$ and $g(x)=e^{-x}+2$ be two functions. Evaluate $(g\ o\ f)^{-1}(3)$.

SOLUTION

$$f(x)=\frac{ax+2}{3x-4}, \mathrm{x} \neq \frac{4}{3}$$

$$(f\ o\ f)(\mathrm{x})=\mathrm{x}$$

$$f[f(\mathrm{x})]=\mathrm{x}$$

$$f\left[\frac{ax+2}{3x-4}\right]=\mathrm{x}$$

$$\frac{a\left[\frac{ax+2}{3x-4}\right]+2}{3\left[\frac{ax+2}{3x-4}\right]-4}=\mathrm{x}$$

$$a\left[\frac{ax+2}{3x-4}\right]+2=\mathrm{x}\left[3\left[\frac{ax+2}{3x-4}\right]-4\right]$$

$$\left[\frac{a^2x+2a}{3x-4}\right]+\left[\frac{2(3x-4)}{3x-4}\right]=\left[\frac{3ax^2+6x}{3x-4}\right]-4x$$

$$\left[\frac{(a^2x+2a)+2(3x-4)}{3x-4}\right]=\left[\frac{3ax^2+6x}{3x-4}\right]-\left[\frac{4x(3x-4)}{3x-4}\right]$$

$$\left[\frac{a^2x+2a+6x-8}{3x-4}\right]=\left[\frac{3ax^2+6x-12x^2+16x}{3x-4}\right]$$

$$a^2x+2a+6x-8=3ax^2+6x-12x^2+16x$$

$$3ax^2-12x^2+6x+16x-a^2x-6x-2a+8=0$$

$$(3a-12)x^2+(16-a^2)x+8-2a=0$$

$$Compare\ coefficient\ of\ x^2$$

$$3a-12=0$$

$$a=4$$

(b)

$$f(x) = ln|3x + 2|$$

$$g(x) = e^{-x} + 2$$

$$g\ o\ f(\mathrm{x}) = g[f(x)]$$

$$= g[ln|3x + 2|]$$

$$= e^{-ln|3x+2|} + 2$$

$$= e^{ln|3x+2|^{-1}} + 2$$

$$= |3x + 2|^{-1} + 2$$

$$= \frac{1}{3x + 2} + 2$$

$$= \frac{1 + 2(3x + 2)}{3x + 2}$$

$$= \frac{1 + 6x + 4}{3x + 2}$$

$$= \frac{6x + 5}{3x + 2}$$

Let $y = \frac{6x+5}{3x+2}$

$$y(3x + 2) = 6x + 5$$

$$3xy + 2y = 6x + 5$$

$$3xy - 6x = 5 - 2y$$

$$x(3y - 6) = 5 - 2y$$

$$x = \frac{5 - 2y}{3y - 6}$$

$$(g\ o\ f)^{-1}(\mathrm{y}) = \frac{5 - 2y}{3y - 6}$$

$$(g\ o\ f)^{-1}(\mathrm{x}) = \frac{5 - 2x}{3x - 6}$$

$$(g\ o\ f)^{-1}(3) = \frac{5 - 2(3)}{3(3) - 6} = -\frac{1}{3}$$

10. (a) Solve the inequality $\left|\frac{x-1}{x+3}\right| > 2$.

(b) Show that $\frac{2^x \text{ x } 4^{2x}}{8^x} = 2^{2x}$.

Hence, find the interval for x so that $\frac{2^x \text{ x } 4^{2x}}{8^x} - 13(2^x) + 36 \geq 0$.

SOLUTION

(a)

$$\left|\frac{x-1}{x+3}\right| > 2$$

$$\frac{x-1}{x+3} > 2$$

$$\frac{x-1}{x+3} - 2 > 0$$

$$\frac{x-1}{x+3} - \frac{2(x+3)}{x+3} > 0$$

$$\frac{x-1-2x-6}{x+3} > 0$$

$$\frac{-x-7}{x+3} > 0$$

Let $-x-7$=0 ➔ $x = -7$

$x + 3 = 0$ ➔ $x = -3$

	$(-\infty, -7)$	$(-7, -3)$	$(-3, \infty)$
$-x-7$	+	-	-
$x+3$	-	-	+
	-	+	-

$(-7, -3)$

Or

$$\frac{x-1}{x+3} < -2$$

$$\frac{x-1}{x+3} + 2 < 0$$

$$\frac{x-1}{x+3} + \frac{2(x+3)}{x+3} < 0$$

$$\frac{x-1+2(x+3)}{x+3} < 0$$

$$\frac{x-1+2x+6}{x+3} < 0$$

$$\frac{3x+5}{x+3} < 0$$

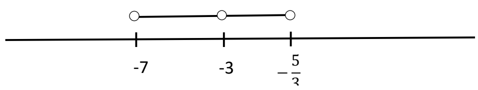
Let $3x+5$=0 ➔ $x = -\frac{5}{3}$

$x + 3 = 0$ ➔ $x = -3$

	$(-\infty, -3)$	$\left(-3, -\frac{5}{3}\right)$	$\left(-\frac{5}{3}, \infty\right)$
$3x+5$	-	-	+
$x+3$	-	+	+
	+	-	+

or

$\left(-3, -\frac{5}{3}\right)$



Solution Interval: $(-7, -3) \cup \left(-3, -\frac{5}{3}\right)$

(b)

Show that $\frac{2^x \times 4^{2x}}{8^x} = 2^{2x}$

$$\frac{2^x \times 4^{2x}}{8^x} = \frac{2^x \ (2^2)^{2x}}{(2^3)^x}$$

$$= \frac{2^x \ 2^{4x}}{2^{3x}}$$

$$= \frac{2^{5x}}{2^{3x}}$$

$$= 2^{2x}$$

$$\frac{2^x \times 4^{2x}}{8^x} - 13(2^x) + 36 \geq 0$$

$$2^{2x} - 13(2^x) + 36 \geq 0$$

$$(2^x)^2 - 13(2^x) + 36 \geq 0$$

$Let\ u = 2^x$

$$(2^x)^2 - 13(2^x) + 36 \geq 0$$

$$u^2 - 13u + 36 \geq 0$$

$$(u-9)(u-4) \geq 0$$

Let $(u-9) = 0$ $\qquad$ $(u-4) = 0$

$\mathrm{u} = 9$ $\qquad$ $\mathrm{u} = 4$

	$(-\infty, 4)$	$(4, 9)$	$(9, \infty)$
$(u-9)$	$-$	$-$	$+$
$(u-4)$	$-$	$+$	$+$
$(u-9)(u-4)$	$+$	$-$	$+$

$u \leq 4$ or $u \geq 9$

$2^x \leq 4$ or $2^x \geq 9$

$\ln 2^x \leq \ln 4$ or $\ln 2^x \geq \ln 9$

$x \ln 2 \leq \ln 4$ or $x \ln 2 \geq \ln 9$

$x \leq \frac{\ln 4}{\ln 2}$ or $x \geq \frac{\ln 9}{\ln 2}$

$x \leq 2$ or $x \geq 3.170$

Solution interval : $(-\infty, 2] \cup [3.17, \infty)$