KEMENTERIAN
PENDIDIKAN
MALAYSIA

QS 025

Mid-Semester Examination

Semester II

Session 2015/2016

Kang Kooi Wei

1. Using integration by parts, evaluate $\int x \cos 3x \, dx$.
2. Find
   a. $\int \frac{5-x}{\sqrt{x}} dx$
   b. $\int \frac{\ln x}{x} dx$
3. Solve the differntial equation $3x^2 \frac{dy}{dx} = (x^2 - 1)\sqrt{y}$. Express y in terms of x.
4. Show that the equation $2x^4 = 5 + x$ has a root between $x = 1$ and $x = 2$. By taking $x = 1.4$ as the first approximation, evaluate this root to three significant fugures using the Newton-Raphson method.
5. The region R is enclosed by the curve $y = \sqrt{6 - x}$, $x - axis$ and $0 \leq x \leq 6$. Find
   a. The area of the region R.
   b. The volume of the solid generated when the region R is rotated through 2π radians about the $x - axis$.
6. (a) Find the general equation of the circle with centre (3, 3) and radius $\sqrt{10}$. If the straight line 3x – y + 4 = 0 is atangent to the circle at point P, find the coordinate of P. Hence, find the equation of the normal at P.

   (b) Express the equation of a parabola $x^2 - 4x - 24y + 76 = 0$ in standard form. Hence, determine the coordinates of its vertex and focus.

END OF QUESTION PAPER

1. Using integration by parts, evaluate $\int x \cos 3x \, dx$.

SOLUTION

$$\int x \cos 3x \, dx$$

$u = x$ $\qquad$ $dv = \cos 3x \;\, dx$

$\frac{du}{dx} = 1$ $\qquad$ $\int dv = \int \cos 3x \;\, dx$

$u = dx$ $\qquad$ $v = \frac{\sin 3x}{3}$

$$\int u \;\, dv = uv - \int v \;\, du$$

$\int x \cos 3x \, dx = (x)\left(\frac{\sin 3x}{3}\right) - \int \frac{\sin 3x}{3} \;\, du$

$= \frac{x \sin 3x}{3} - \frac{1}{3}\int \sin 3x \;\, du$

$= \frac{x \sin 3x}{3} - \frac{1}{3}\left(\frac{-\cos 3x}{3}\right) + c$

$= \frac{x \sin 3x}{3} + \frac{\cos 3x}{9} + c$

2. Find

a. $\int \frac{5-x}{\sqrt{x}}\,dx$

b. $\int \frac{\ln x}{x}\,dx$

SOLUTION

a. $\int \frac{5-x}{\sqrt{x}}\,dx$ $\int \frac{5-x}{\sqrt{x}}\,dx = \int \frac{5}{\sqrt{x}}\,dx - \int \frac{x}{\sqrt{x}}\,dx$ $= 5\int x^{-\frac{1}{2}}\,dx - \int x^{\frac{1}{2}}\,dx$ $= 5\left(2x^{\frac{1}{2}}\right) - \frac{2x^{\frac{3}{2}}}{3} + c$ $= 10x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c$	**a.** $\int \frac{5-x}{\sqrt{x}}\,dx$ $u = \sqrt{x}$ $u^2 = x$ $2u\frac{du}{dx} = 1$ $2udu = dx$ $\int \frac{5-x}{\sqrt{x}}\,dx = \int \frac{5-u^2}{u}(2u\,du)$ $= 2\int 5 - u^2\,du$ $= 2\left[5u - \frac{u^3}{3}\right] + c$ $= 2\left[5\sqrt{x} - \frac{\sqrt{x}^3}{3}\right] + c$ $= 10\sqrt{x} - \frac{\sqrt{x}^3}{3} + c$

b. $\int \frac{\ln x}{x} dx$ $u = \ln x$ $\frac{du}{dx} = \frac{1}{x}$ $xdu = dx$ $\int \frac{\ln x}{x} dx = \int \frac{u}{x}(x\,du)$ $= \int u\,du$ $= \frac{u^2}{2} + C$ $= \frac{(\ln x)^2}{2} + C$	**b.** $\int \frac{\ln x}{x} dx$ $u = \ln x \qquad dv = \frac{1}{x}dx$ $\frac{du}{dx} = \frac{1}{x} \qquad v = \int \frac{1}{x}dx$ $du = \frac{1}{x}\,dx \qquad v = \ln x$ $\int \frac{\ln x}{x} dx = uv - \int v\,du$ $\int \frac{\ln x}{x} dx = (\ln x)(\ln x) - \int (\ln x)\left(\frac{1}{x}\,dx\right)$ $\int \frac{\ln x}{x} dx = (\ln x)^2 - \int \frac{\ln x}{x} dx$ $\int \frac{\ln x}{x} dx + \int \frac{\ln x}{x} dx = (\ln x)^2$ $2\int \frac{\ln x}{x} dx = (\ln x)^2 + c$ $\int \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2} + c$

3. Solve the differntial equation $3x^2\frac{dy}{dx}=(x^2-1)\sqrt{y}$. Express y in terms of x.

SOLUTION

$$3x^2\frac{dy}{dx}=(x^2-1)\sqrt{y}$$

$$\int\frac{1}{\sqrt{y}}dy=\int\frac{(x^2-1)}{3x^2}dx$$

$$\int y^{-\frac{1}{2}}dy=\int\frac{x^2}{3x^2}-\frac{1}{3x^2}dx$$

$$\int y^{-\frac{1}{2}}dy=\int\frac{1}{3}-\frac{1}{3}x^{-2}dx$$

$$\frac{y^{\frac{1}{2}}}{\frac{1}{2}}=\frac{1}{3}x-\frac{1}{3}\left(\frac{x^{-1}}{-1}\right)+c$$

$$2y^{\frac{1}{2}}=\frac{1}{3}x+\frac{1}{3x}+c$$

$$y^{\frac{1}{2}}=\frac{1}{6}x+\frac{1}{6x}+c$$

$$y=\left(\frac{1}{6}x+\frac{1}{6x}+c\right)^2$$

4. Show that the equation $2x^4 = 5 + x$ has a root between $x = 1$ and $x = 2$. By taking $x = 1.4$ as the first approximation, evaluate this root to three significant fugures using the Newton-Raphson method.

SOLUTION

Let $f(x) = 2x^4 - x - 5$

$f(1) = 2(1)^4 - (1) - 5 = -4 < 0$

$f(2) = 2(2)^4 - (2) - 5 = 25 > 0$

Since $f(1)$ and $f(2)$ have different signs, therefore $2x^4 = 5 + x$ has a root between $x = 1$ and $x = 2$.

$f'(x) = 8x^3 - 1$

$x_0 = 1.4$

$x_1 = 1.4 - \frac{f(1.4)}{f'(1.4)} = 1.4 - \frac{2(1.4)^4-(1.4)-5}{8(1.4)^3-1}$=1.33876

$x_2 = 1.33876 - \frac{f(1.33876)}{f'(1.33876)} = 1.33876 - \frac{2(1.33876)^4-(1.33876)-5}{8(1.33876)^3-1}$=1.334046

$x_3 = 1.334046 - \frac{f(1.334046)}{f'(1.334046)} = 1.33876 - \frac{2(1.334046)^4-(1.334046)-5}{8(1.334046)^3-1}$=1.33402

$x_4 = 1.33402 - \frac{f(1.33402)}{f'(1.33402)} = 1.33876 - \frac{2(1.33402)^4-(1.33402)-5}{8(1.33402)^3-1}$=1.33402

$x_4 = 1.33402 - \frac{f(1.33402)}{f'(1.33402)} = 1.33876 - \frac{2(1.33402)^4-(1.33402)-5}{8(1.33402)^3-1}$=1.33402

$\therefore The\ root\ is\ 1.33$

5. The region R is enclosed by the curve $y = \sqrt{6-x}$, $x - axis$ and $0 \leq x \leq 6$. Find
   a. The area of the region R.
   b. The volume of the solid generated when the region R is rotated through 2π radians about the $x - axis$.

SOLUTION

5(a) $Area = \int_0^6 \sqrt{6-x}\ dx$

$$= \int_0^6 (6-x)^{\frac{1}{2}}\ dx$$

$$= \left[\frac{(6-x)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)(-1)}\right]_0^6$$

$$= -\frac{2}{3}\left[(6-x)^{\frac{3}{2}}\right]_0^6$$

$$= -\frac{2}{3}\left[(6-6)^{\frac{3}{2}} - (6-0)^{\frac{3}{2}}\right]$$

$$= -\frac{2}{3}\left[0 - (6)^{\frac{3}{2}}\right]$$

$$= 9.798 \quad @ \quad 4\sqrt{6}\ unit^2$$

5(b) $Volume = \pi \int_0^6 \left(\sqrt{6-x}\right)^2\ dx$

$$= \pi \int_0^6 6 - x\ dx$$

$$= \pi \left[6x - \frac{x^2}{2}\right]_0^6$$

$$= \pi \left[\left(6(6) - \frac{6^2}{2}\right) - \left(6(0) - \frac{0^2}{2}\right)\right]$$

$$= \pi[(36-18) - (0-0)]$$

$$= 18\pi\ unit^3$$

6. (a) Find the general equation of the circle with centre (3, 3) and radius $\sqrt{10}$. If the straight line 3x – y + 4 = 0 is atangent to the circle at point P, find the coordinate of P. Hence, find the equation of the normal at P.

(b) Express the equation of a parabola $x^2 - 4x - 24y + 76 = 0$ in standard form. Hence, determine the coordinates of its vertex and focus.

SOLUTION

6(a) Centre, $(h,k) = (3,3)$

Radius, $r = \sqrt{10}$

$(x-h)^2 + (y-k)^2 = (r)^2$

$(x-3)^2 + (y-3)^2 = \left(\sqrt{10}\right)^2$

$(x^2 - 6x + 9) + (y^2 - 6y + 9) = 10$

$x^2 + y^2 - 6x - 6y + 18 = 10$

$x^2 + y^2 - 6x - 6y + 8 = 0$ .... (1)

$Substitute\ y = 3x + 4\ into\ (1)$

$x^2 + (3x+4)^2 - 6x - 6(3x+4) + 8 = 0$

$x^2 + 9x^2 + 24x + 16 - 6x - 18x - 24 + 8 = 0$

$10x^2 = 0$

$x = 0$

$y = 3x + 4 = 3(0) + 4 = 4$

$Point\ P = (0,4)$

$M_T = 3$

$M_n = -\frac{1}{3}$

Equations of Normal

$y - 4 = -\frac{1}{3}(x - 0)$

$$y = -\frac{1}{3}x + 4$$

$$x + 3y - 12 = 0$$

6(b) $x^2 - 4x - 24y + 76 = 0$

$$\left[x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2\right] - 24y + 76 = 0$$

$$[(x-2)^2 - 4] - 24y + 76 = 0$$

$$(x-2)^2 = 24y - 72$$

$$(x-2)^2 = 24(y-3)$$

$$(x-2)^2 = 4(6)(y-3)$$

$$\therefore V(h,k) = V(2,3)$$

$$4p = 24$$

$$p = 6$$

$$F(h, k+p) = F(2, 9)$$