

QS026
Mathematics
Semester 2
2007/2008
1 hour



QS026
Matematik
Semester II
2007/2008
1 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

UJIAN PERTENGAHAN SEMESTER PROGRAM MATRIKULASI
MID-SEMESTER EXAMINATION

MATEMATIK
1 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

ARAHAN KEPADA CALON:

Kertas soalan ini mengandungi 7 soalan.

Jawab **semua** soalan.

Markah penuh yang diperuntukkan bagi tiap-tiap soalan atau bahagian soalan ditunjukkan dalam kurungan pada penghujung soalan atau bahagian soalan.

Semua langkah kerja hendaklah ditunjukkan dengan jelas.

Kalkulator saintifik yang tidak boleh diprogramkan sahaja boleh digunakan.

Jawapan berangka boleh diberi dalam bentuk π , e, surd, pecahan atau sehingga tiga angka bererti, di mana-mana yang sesuai, kecuali jika dinyatakan dalam soalan.

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of 7 questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of each of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers can be given in the form of π , e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

Kertas soalan ini mengandungi 5 halaman bercetak.

This booklet consists of 5 printed pages.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

Hyperbolic

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\operatorname{coth}^2 x - 1 = \operatorname{cosech}^2 x$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

Differentiation and Integration

$f(x)$	$f'(x)$
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$\cot x$	$-\operatorname{cosec}^2 x$
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$\sec x$	$\sec x \tan x$
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$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
--------------------------	----------------------------------

$f(x)$	$f'(x)$
--------	---------

$\operatorname{coth} x$	$-\operatorname{cosech}^2 x$
-------------------------	------------------------------

$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
-------------------------	----------------------------------

$\operatorname{cosech} x$	$-\operatorname{cosech} x \operatorname{coth} x$
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$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

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1. Express the equation of parabola $y^2 + 4y - 12x - 8 = 0$ in the standard form.

Hence, determine the vertex and the focus of the parabola.

[5 marks]

2. Prove that

$$\frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} = \cos \theta + \sin \theta.$$

[5 marks]

3. Given the parametric equations $x = 4\cos^2 t$ and $y = 5\sin t$.

Find $\frac{dy}{dx}$ when $t = \frac{\pi}{6}$.

[6 marks]

4. Solve the following.

(a) $\int \cosh^2 x \sinh x \, dx$.

[3 marks]

(b) $\int x \cosh 2x \, dx$.

[3 marks]

5. Find the equation of the normal to the curve $9x^3 + y^3 = 4y^2$ at $y = 3$.

[7 marks]

6. Express $\sin 2\theta - \cos 2\theta$ in the form $R \sin(2\theta - \alpha)$ where $R > 0$ and

$0^\circ \leq \alpha \leq 90^\circ$. Hence, solve the equation $\sin 2\theta - \cos 2\theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$.

[10 marks]

7. A circle touches the line $5x + y = 3$ at the point $(2, -7)$ and its centre lies on the line $x - 2y = 19$. Find the point of intersection between the normal to the circle at $(2, -7)$ and the line $x - 2y = 19$. Hence, determine the centre and the standard equation of the circle.

[11 marks]

END OF QUESTION