QS026
Mathematics
Semester II
2009/2010
1 hour

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Matematik
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1 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

UJIAN PERTENGAHAN SEMESTER PROGRAM MATRIKULASI
MID-SEMESTER EXAMINATION

MATEMATIK
1 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

ARAHAN KEPADA CALON:

Kertas soalan ini mengandungi **7** soalan.

Jawab **semua** soalan.

Markah penuh yang diperuntukkan bagi tiap-tiap soalan atau bahagian soalan ditunjukkan dalam kurungan pada penghujung soalan atau bahagian soalan.

Semua langkah kerja hendaklah ditunjukkan dengan jelas.

Kalkulator saintifik yang tidak boleh diprogramkan sahaja boleh digunakan.

Jawapan berangka boleh diberi dalam bentuk π, e, surd, pecahan atau sehingga tiga angka bererti, di mana-mana yang sesuai, kecuali jika dinyatakan dalam soalan.

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **7** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of each of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers can be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

Hyperbolic

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$
$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$1 - \tanh^2 x = \operatorname{sech}^2 x$$
$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$
$$\sinh 2x = 2 \sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

Differentiation and Integration

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$f(x)$	$f'(x)$
$\coth x$	$-\operatorname{cosech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c$$

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1. Express the equation of a hyperbola $4x^2 - y^2 - 8x + 4y - 4 = 0$ in standard form. Hence, determine the coordinates of the centre and the vertices.

   [6 *marks*]

2. Solve the equation $4\tan^2 x - 6\tan x + 2 = 0$ for $0° \le x \le 360°$.

   [5 *marks*]

3. Show that $\dfrac{\sin 2\theta + \sin \theta}{\cos 2\theta + \cos \theta + 1} = \tan \theta.$

   [5 *marks*]

4. A curve is given by the parametric equations $x = 2t^2 + 3$, $y = 2t + 1$. Find the equation of the normal to the curve for $t = 1$.

   [7 *marks*]

5. a) By using the definition of hyperbolic function, show that

   $$\frac{d}{dx}(\sinh x) = \cosh x.$$

   b) Find $\int x\cosh x^2 dx$.

   [7 *marks*]

6. If $\tan\dfrac{x}{2} = t$, express $\tan x$, $\sin x$ and $\cos x$ in terms of t.

   Hence, solve the equation $4\sin x - 3\cos x = 4$ for $0° \le x \le 360°$.

   [10 *marks*]

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7. The equation of circle P is given by

$$x^2 + y^2 - 4x + 6y - 12 = 0.$$

    a)   Find the coordinates of its center and radius.

    b)   Find the perpendicular distance from the center of P to the line $3x + 4y = k$ in terms of k, where k is a constant.

    c)   Hence, find the values of k such that the line $3x + 4y = k$ is a tangent to P.

[10 *marks*]

END OF QUESTION BOOKLET

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